回転・遠心力

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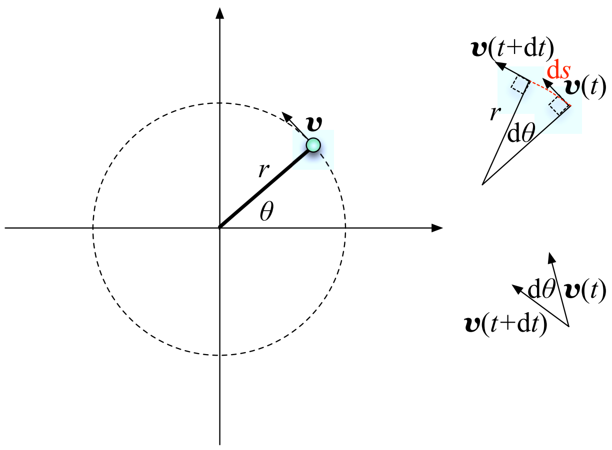


We consider the weight connected to a string is rotating around the origin at a constant velocity v. The velocity v is always constant, so the force should be perpendicular to the velocity. The force is given by the the string tension. The acceleration $\mathbf{a}$ is given by

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \lim_{\mathrm{d}t \to 0} \frac{\mathbf{v}(t + \mathrm{d}t) - \mathbf{v}(t)}{\mathrm{d}t} \tag{1}$$

Then the magnitude of the acceleration a is

$$a = \frac{v\mathrm{d}\theta}{\mathrm{d}t} = v\frac{\mathrm{d}\theta}{\mathrm{d}t} = v\omega \tag{2}$$

The length of the arc ds is given by

$$\mathrm{d}s = r\mathrm{d}\theta \tag{3}$$

The velocity v of the weight is goven by

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = r\frac{\mathrm{d}\theta}{\mathrm{d}t} = r\omega \tag{4}$$

The acceleration a is given by

$$a = v\omega = r\omega^2 \tag{5}$$

The force that is perpendicular to the weight motion does not affect on the motion, then the Newton equation becomes

$$F = ma = mr\omega^2 \tag{6}$$

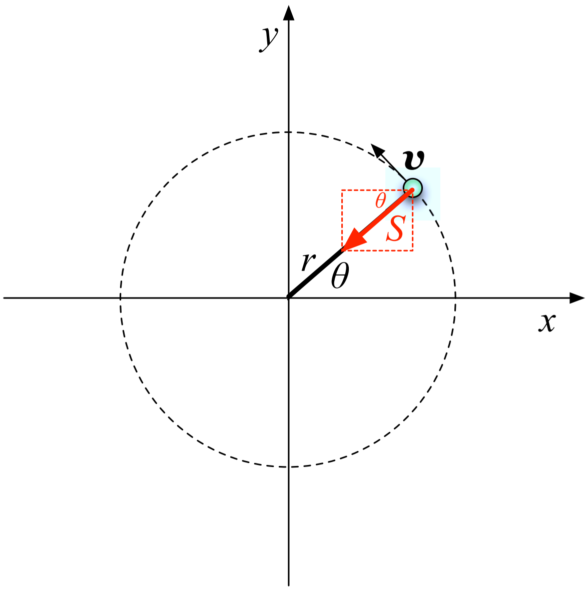
The motion given by

$$\theta = \omega t \tag{7}$$

Another solution: The tension of the string is given by $S = mr\omega^2$. The equations of motion in the x and y direction are given by

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -S\cos\theta, \quad x = r\cos\theta \tag{8}$$

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -S\sin\theta, \quad y = r\sin\theta \tag{9}$$



We can solve the equation of motion in x and y direction.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{S}{mr}x = -\omega^2 x, \quad x = x_0\cos(\omega t) \tag{10}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -\frac{S}{mr}y = -\omega^2 y, \quad y = y_0\sin(\omega t) \tag{11}$$

There is one more condition

$$x^2 + y^2 = r^2 \tag{12}$$

$$x_0^2\cos^2(\omega t) + y_0^2\sin^2(\omega t) = r^2 \tag{13}$$

$$x_0 = y_0 = r \tag{14}$$