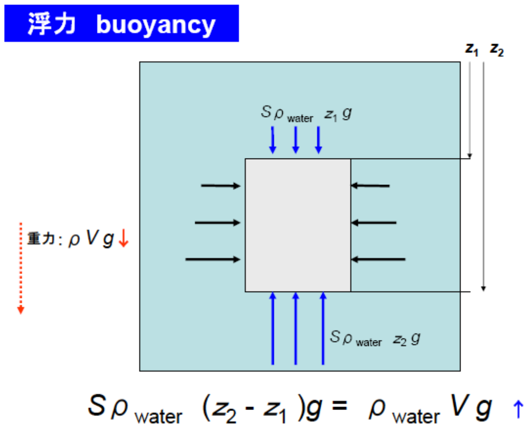
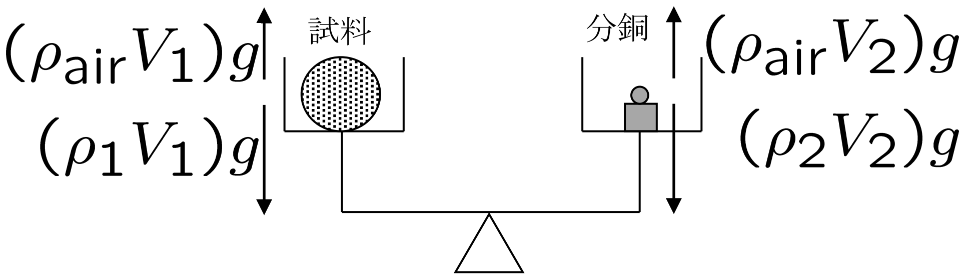
浮力(buoyancy)とは？　MY 2016March11



今，水中に円筒状のものを沈めたとしよう。筒の側面にかかる静水圧は円筒の円周上ですべてキャンセルされるので総和はゼロである。問題は筒の上面と低面である。上面にかかる静水圧は下面にかかる静水圧より小さいのでこの差が浮力となる。具体に式で求めてみよう。筒の上面・底面の面積を S とする。また水面から上面までの距離を z_1,水面から底面までの距離を z_2 とする。従って，上面にかかる力は水面から上面までに存在する水がつくる圧に面積をかけたものになり $S\rho_{water}z_1g$ となる。ここで,ρ_{water} は水の密度,g は重力加速度である。同じく底面に働く力は，$S\rho_{water}z_2g$ となる。その差 $S\rho_{water}z_2g$ - $S\rho_{water}z_1g = \rho_{water}Vg$ が浮力である。ここで V は筒の体積である。筒の密度を ρ とすると，重力は下方に ρVg，浮力は上方に $\rho_{water}Vg$ かかるので，あわせると下方に$(\rho - \rho_{water})Vg$ の力がかかることになる。

天秤の浮力補正の模式図および近似式



$$\rho_1V_1g - \rho_{air}V_1g = \rho_2V_2g - \rho_{air}V_2g$$
$$\rho_1V_1\left(1-\frac{\rho_{air}V_1}{\rho_1V_1}\right) = \rho_2V_2\left(1-\frac{\rho_{air}V_2}{\rho_2V_2}\right)$$
$$W_1\left(1-\frac{\rho_{air}}{\rho_1}\right) = W_2\left(1-\frac{\rho_{air}}{\rho_2}\right)$$
$$W_1 = W_2\frac{1-\frac{\rho_{air}}{\rho_2}}{1-\frac{\rho_{air}}{\rho_1}} \qquad \text{(3-1)}$$

試料の質量：$W_1 = \rho_1V_1$

分銅の質量：$W_2 = \rho_2V_2$

$$W_1 \simeq W_2\left(1-\frac{\rho_{air}}{\rho_2}\right)\left(1+\frac{\rho_{air}}{\rho_1}\right)$$
$$\simeq W_2\left(1+\frac{\rho_{air}}{\rho_1}-\frac{\rho_{air}}{\rho_2}\right) \qquad \text{(2-1)}$$

$$V_{20}\{1+3\gamma(t-20)\} = \frac{W_1}{\rho_1} = W_2\frac{1-\frac{\rho_{air}}{\rho_2}}{\rho_1-\rho_{air}}$$
$$V_{20} \simeq W_2\frac{1}{\rho_1-\rho_{air}}\left(1-\frac{\rho_{air}}{\rho_2}\right)\{1-3\gamma(t-20)\} \qquad \text{(3-4)}$$

V_{20}:ガラス測容器の 20℃での体積

γ:ガラスの線膨張係数

t:水温

導出の際,以下の関係を用いた。 $\frac{1}{1+x} \simeq 1-x+....,\quad x << 1$

Taylor展開：$f(x) \simeq f(0)+f'(0)x+(1/2!)f''(0)x^2+\ldots$

Bouyancy: sphere

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The bouyancy is easily derived from the cylinder. The pressure from water (environment), which depends on the depth from the water surface, are cancelled out at the cylinder side and the pressure difference between the lower and upper surface contribute to the bouyancy.

How about sphere? The sphere with density ρ is immersed in the liquid with the density $\rho_{\rm env}$. The liquid is existed $z \geq 0$ and the center of the sphere is located at $z = z_0$. The radius of the sphere is r.

Let's consider the circular ring on the sphere. The ring is located at $z = z_0 + r\cos\theta$ with the radius $R = r\sin\theta$. The angle θ is measured from the z-axis which range $0 \leq \theta \leq \pi$ and is shown in the Fig.1. The pressure or force acts on the spherical surface normal direction. The magnitude of the force $\mathrm{d}F$ on the infinitesimal surface area $\mathrm{d}S = 2\pi R r \mathrm{d}\theta$ is given by

$$\mathrm{d}F = \rho_{\rm env} g z \mathrm{d}S = \rho_{\rm env} g (z_0 + r\cos\theta) 2\pi R r \mathrm{d}\theta = \rho_{\rm env} g (z_0 + r\cos\theta) 2\pi r^2 \sin\theta \mathrm{d}\theta$$

The forces in the R-direction are cancelled out and we should consider the z-direction. The force in the z-direction is given by

$$\mathrm{d}F_z = \mathrm{d}F\cos\theta = \rho_{\rm env} g (z_0 + r\cos\theta) 2\pi r^2 \sin\theta\cos\theta \mathrm{d}\theta$$

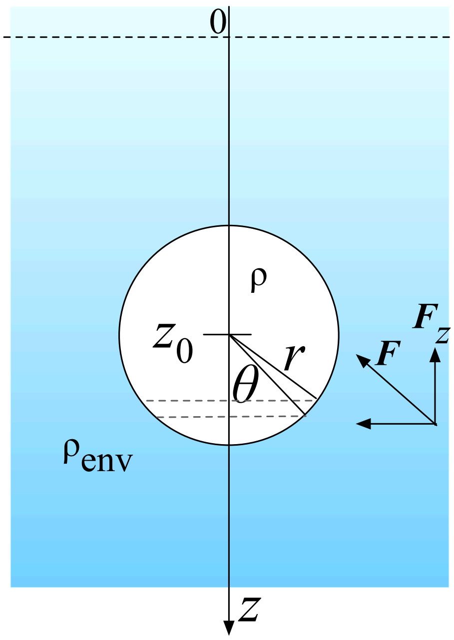

Figure 1: The sphere with density ρ is immersed in the liquid with the density $\rho_{\rm env}$. The liquid is existed $z \geq 0$ and the center of the sphere is located at $z = z_0$. The radius of the sphere is r.

If we integreate θ from 0 to π

$$F_z = \rho_{\rm env} g \int_0^{\pi} \mathrm{d}\theta (z_0 + r\cos\theta) 2\pi r^2 \sin\theta\cos\theta$$

$$= \rho_{\mathrm{env}} g 2\pi r^2 z_0 \int_0^{\pi} \mathrm{d}\theta \sin\theta\cos\theta + \rho_{\mathrm{env}} g 2\pi r^3 \int_0^{\pi} \mathrm{d}\theta \sin\theta\cos^2\theta$$

If we put $X = \cos\theta$

$$\begin{aligned}
\mathrm{d}X &= -\sin\theta\mathrm{d}\theta, \quad (\theta = 0 \to X = 1, \theta = \pi \to X = -1) \\
F_z &= \rho_{\mathrm{env}} g 2\pi r^2 z_0 \int_1^{-1} \mathrm{d}X(-1)X + \rho_{\mathrm{env}} g 2\pi r^3 \int_1^{-1} \mathrm{d}X(-1)X^2 \\
&= \rho_{\mathrm{env}} g 2\pi r^2 z_0 \left[-\frac{1}{2}X^2\right]_1^{-1} + \rho_{\mathrm{env}} g 2\pi r^3 \left[-\frac{1}{3}X^3\right]_1^{-1} \\
&= 0 + \rho_{\mathrm{env}} g \frac{4}{3}\pi r^3 = \rho_{\mathrm{env}} g V_{\mathrm{sphere}}
\end{aligned}$$

Then we get the bouyancy of the sphere is given by $\rho_{\mathrm{env}} g V_{\mathrm{sphere}}$, where V_{sphere} is the volume of the sphere.