Photoelastic Modulator

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If the the light (Electric field in the plane wave form) can pass through the solid with has a complex optical index $n + i\kappa$ and propagate in the z-direction

$$\mathbf{E} = \mathbf{E}_0 e^{i(kz-\omega t)} = \mathbf{E}_0 e^{-\kappa\omega z/c} e^{i(n\omega z/c-\omega t)} \tag{1}$$

Here $k = 2\pi/\lambda = (n + i\kappa)\omega/c$, ω is the angular frequency, c is the speed of light.

In the solid the refractive idex can be described as

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \tag{2}$$

Here x, y, z is the high symmetery direction in the solid. In the fused silica (glass) the solid is isotropic then $n_x = n_y = n_z$.

The dielectric constants (then refractive index) are function not only of applied electric field and of stress of the crystal. In the case of electric field we may write

$$n = n_0 + aE_0 + bE_0^2 + ... \tag{3}$$

Now suppose the crystal has a center of symmetry. If we reverse the elecric field

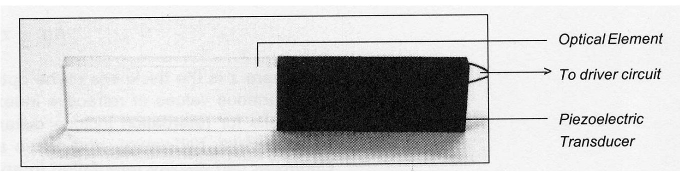
$$n = n_0 - aE_0 + bE_0^2 + ... \tag{4}$$

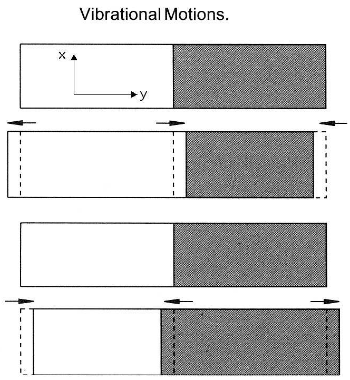
The regractive index should be the same, then $a = 0$.

If a uniaxial stress σ is applied to the crystal axis y, the refractive index in the y-direction becomes

$$n = n_0 + aE_0 + a'\sigma + bE_0^2 + b'\sigma^2 + b''E_0\sigma + ... \tag{5}$$

Even in the centrosymmetric media the change of sign of σ, i.e. tension and compression, induces the change n. This is why even an isotropic material like glass shows a first-order photoelastic effect, but cannot show a first-order electro-optical effect.





If the input light to the elastic modulator is linear-polarized α degree to x-axis and the light is propagates in z-direction.

$$\begin{aligned} \mathbf{E}^{\text{in}}(z=0) &= E_0(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})e^{-i\omega t} \qquad (6)\\ &\qquad \mathbf{i} = (1,0,0), \quad \mathbf{j} = (0,1,0) \\ I^{\text{in}}(z=0) &= \mathbf{E}^{\text{in}*}(z=0)\cdot\mathbf{E}^{\text{in}}(z=0) \\ &= E_0^2(\cos^2\alpha + \sin^2\alpha) = E_0^2 \qquad (7) \end{aligned}$$

If the photoelastic modulater (PEM) is located between $z = 0$ and $z = d$ and the light absortption in the PEM is negligible, the electric field at $z = d$ is given by

$$\begin{aligned}
\mathbf{E}^{\text{out}}(z=d) &= E_0(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}e^{ia'\sigma\omega d/c})e^{i(n\omega d/c - \omega t)} \\
&= E_0(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}e^{i\delta})e^{i(n\omega d/c - \omega t)} \qquad (8) \\
I^{\text{out}}(z=d) &= \mathbf{E}^{\text{out}}(z=d)^*\mathbf{E}^{\text{out}}(z=d) = E_0^2(\cos^2\alpha + \sin^2\alpha) = E_0^2 \qquad (9)
\end{aligned}$$

where the phase shift is defined as $\delta = a'\sigma\omega d/c$. The output polarization is shown in the following figure. In the photoelastic modulator AC-voltage is applied to piezo-oscillator is

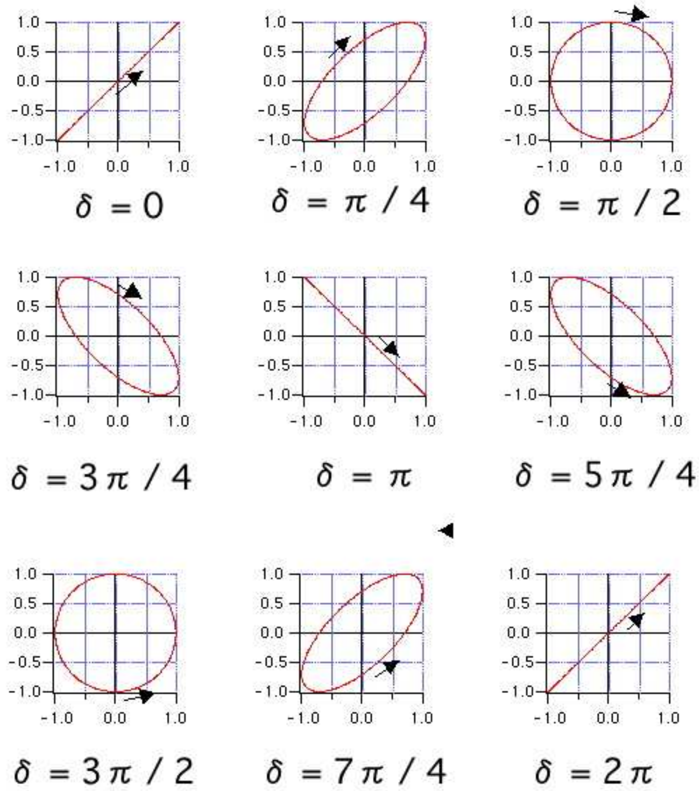


Figure 1: The output of the electric field by photoelastic modulator. $x = \sin\theta, y = \sin(\theta + \delta)$ for $0 \le \theta \le 2\pi$

used,

$$\sigma = V_{\text{piezo}}e^{-i\gamma t} \qquad (10)$$

Then the phase shift is given by

$$\delta(t) = a'\sigma\omega d/c = a'V_{\text{piezo}}e^{-i\gamma t}\omega d/c = AV_{\text{piezo}}e^{-i\gamma t}/\lambda \qquad (11)$$

For the ellipsometer the p-wave is $\mathbf{i}$-direction and $s-$wave is $\mathbf{j}$-direction. For the PM-FTIR the p-wave is $[\mathbf{i}+\mathbf{j}]$-direction and $s-$wave is $[-\mathbf{i}+\mathbf{j}]$-direction. If we can decpompose the $\mathbf{E}_{out}$ to $p-$wave and $s-$wave in the case of PM-FTIR.

$$\begin{aligned}
E_p^{\text{out}} &= E_0(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}e^{i\delta}) \cdot \frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}} = \frac{E_0}{\sqrt{2}}(\cos\alpha + \sin\alpha e^{i\delta}) \qquad (12) \\
I_p^{\text{out}} &= \frac{E_0^2}{2}(\cos^2\alpha + \sin^2\alpha + 2\cos\alpha\sin\alpha\cos\delta) = \frac{E_0^2}{2}(1 + \sin(2\alpha)\cos\delta) \qquad (13) \\
E_s^{\text{out}} &= E_0(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j}e^{i\delta}) \cdot \frac{-\mathbf{i}+\mathbf{j}}{\sqrt{2}} = \frac{E_0}{\sqrt{2}}(-\cos\alpha + \sin\alpha e^{i\delta}) \qquad (14) \\
I_s^{\text{out}} &= \frac{E_0^2}{2}(\cos^2\alpha + \sin^2\alpha - 2\cos\alpha\sin\alpha\cos\delta) = \frac{E_0^2}{2}(1 - \sin(2\alpha)\cos\delta) \qquad (15)
\end{aligned}$$