振り子

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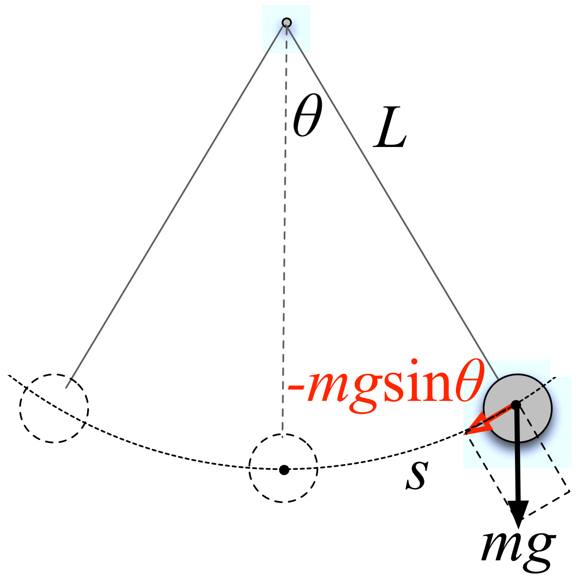


The length of the arc s is given by

$$s = L\theta \tag{1}$$

The velocity v of the weight is defined by

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = L\frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{2}$$

The acceleration a is given by

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = L\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \tag{3}$$

The force that is perpendicular to the weight motion does not affect on the motion, then the Newton equation becomes

$$F = ma, \quad -mg\sin\theta = mL\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \tag{4}$$

If we assume $\theta << 1$, then we can write

$$\sin\theta \simeq \theta - O[\theta^3] \tag{5}$$

$$-mg\theta = mL\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}, \quad \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}\theta \tag{6}$$

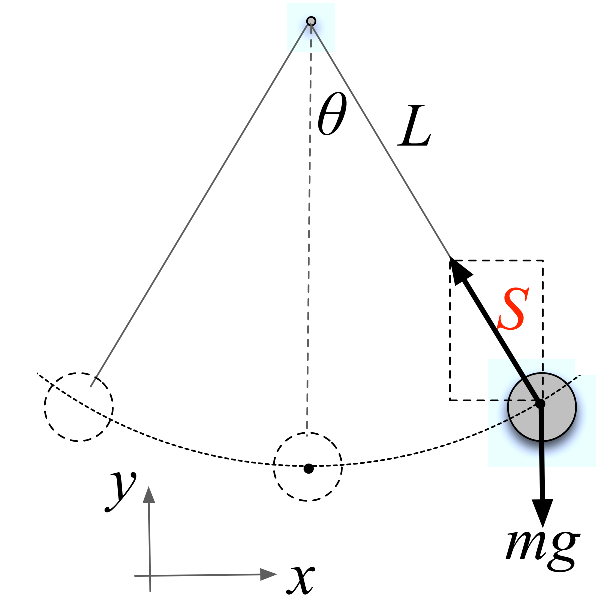
$$\theta = \theta_0 e^{-i\omega t}, \quad -\omega^2\theta = -\frac{g}{L}\theta \tag{7}$$

$$\omega = \sqrt{\frac{g}{L}}, \quad \nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}, \quad T = \frac{1}{\nu} = 2\pi\sqrt{\frac{L}{g}} \tag{8}$$

Another solution: The tension of the string is given by S. The equations of motion in the x and y direction are given by

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -S\sin\theta, \quad x = L\sin\theta \tag{9}$$

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = S\cos\theta - mg, \quad y = L\cos\theta \tag{10}$$



If we assume $\theta << 1$, then we can write

$$\sin\theta \simeq \theta - O[\theta^3], \quad \cos\theta \simeq 1 - O[\theta^2] \tag{11}$$

$$x \simeq L\theta, \quad y \simeq L \tag{12}$$

$$mL\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -S\theta, \quad m\frac{\mathrm{d}^2L}{\mathrm{d}t^2} = S - mg = 0 \tag{13}$$

$$mL\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -mg\theta \tag{14}$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{L}\theta \tag{15}$$