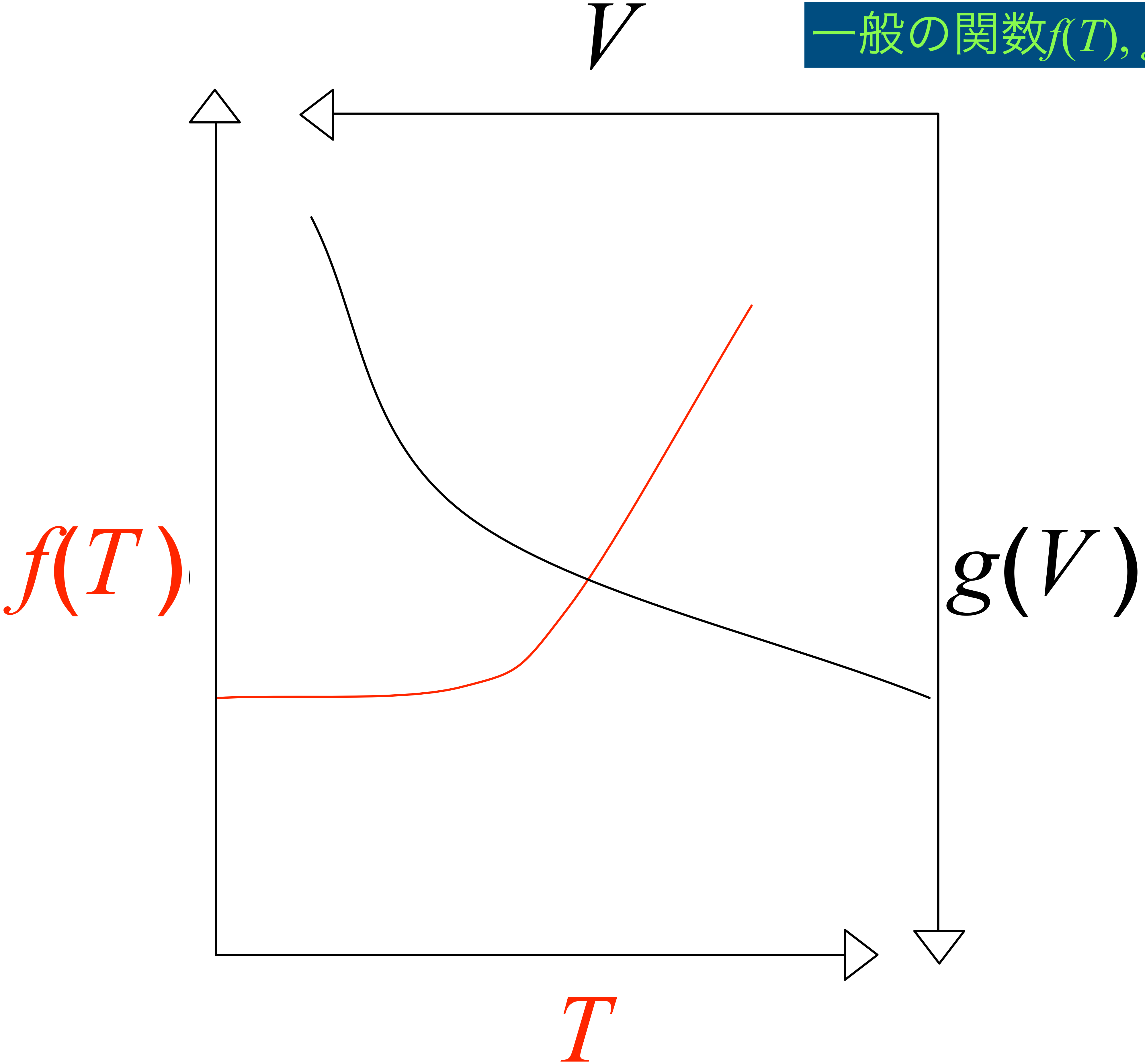
# 変数分離

$$c\frac{\mathrm{d}T}{T} = -\frac{\mathrm{d}V}{V} \qquad (*)$$

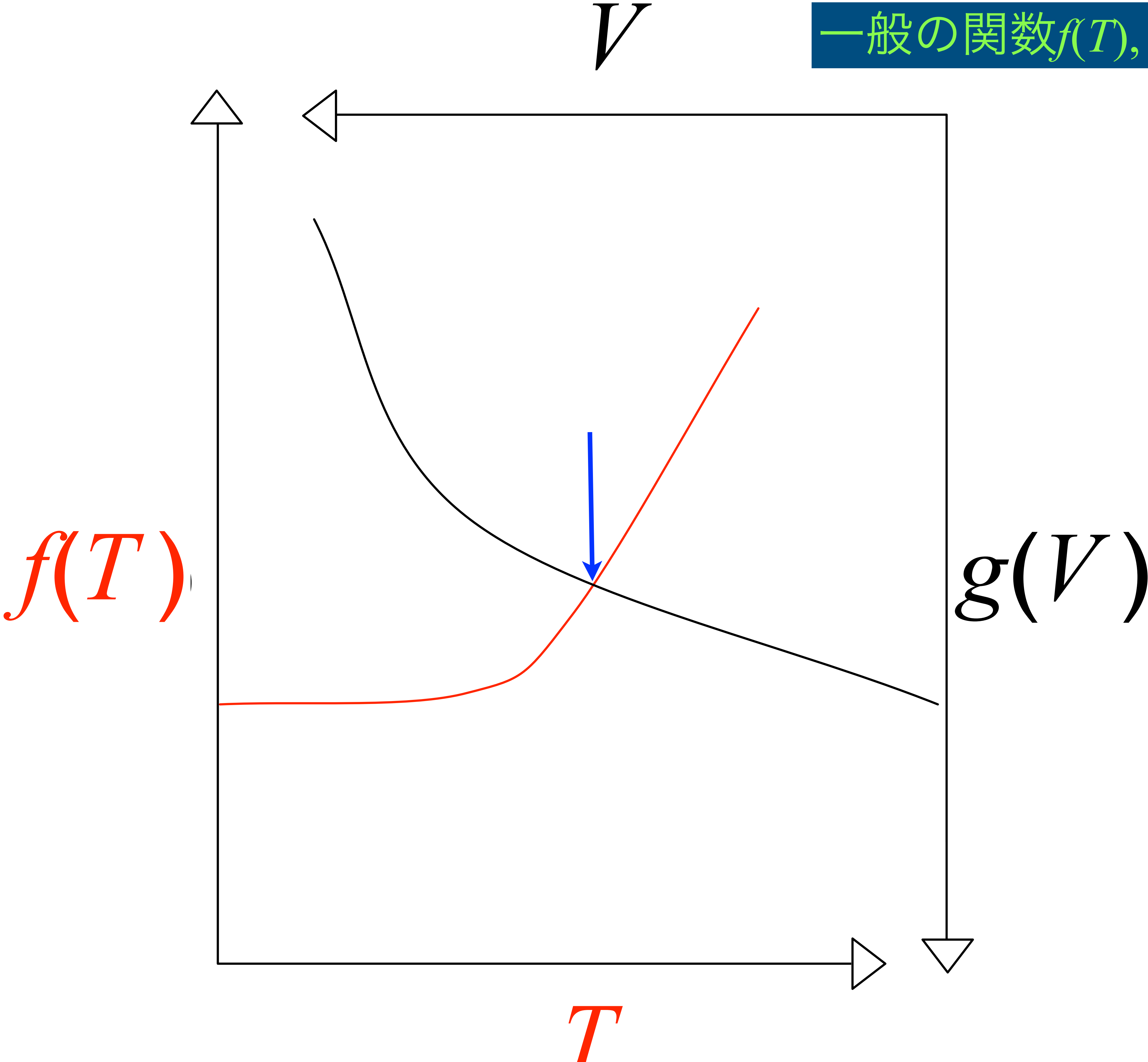
左辺は$T$だけの関数で，右辺は$V$だけの関数である。いかなる$T, V$でも等号が成立するためには，左辺および右辺はある定数で$A$ないといけない。

* 断熱過程では$\mathrm{d}Q = 0$なので$\mathrm{d}U = \mathrm{d}Q(= 0) + \mathrm{d}W = \mathrm{d}W = -P\mathrm{d}V$となり，$\mathrm{d}U = C_V\mathrm{d}T$なので，$\mathrm{d}U = C_V\mathrm{d}T = -P\mathrm{d}V = -nRT / V\,\mathrm{d}V$, 両辺を$nRT$で割って，$(C_V / nR)\,\mathrm{d}T / T = -\mathrm{d}V / V$, $c\,\mathrm{d}T / T = -\mathrm{d}V / V$

一般の関数$f(T)$, $g(V)$の場合



一般の関数$f(T)$, $g(V)$の場合



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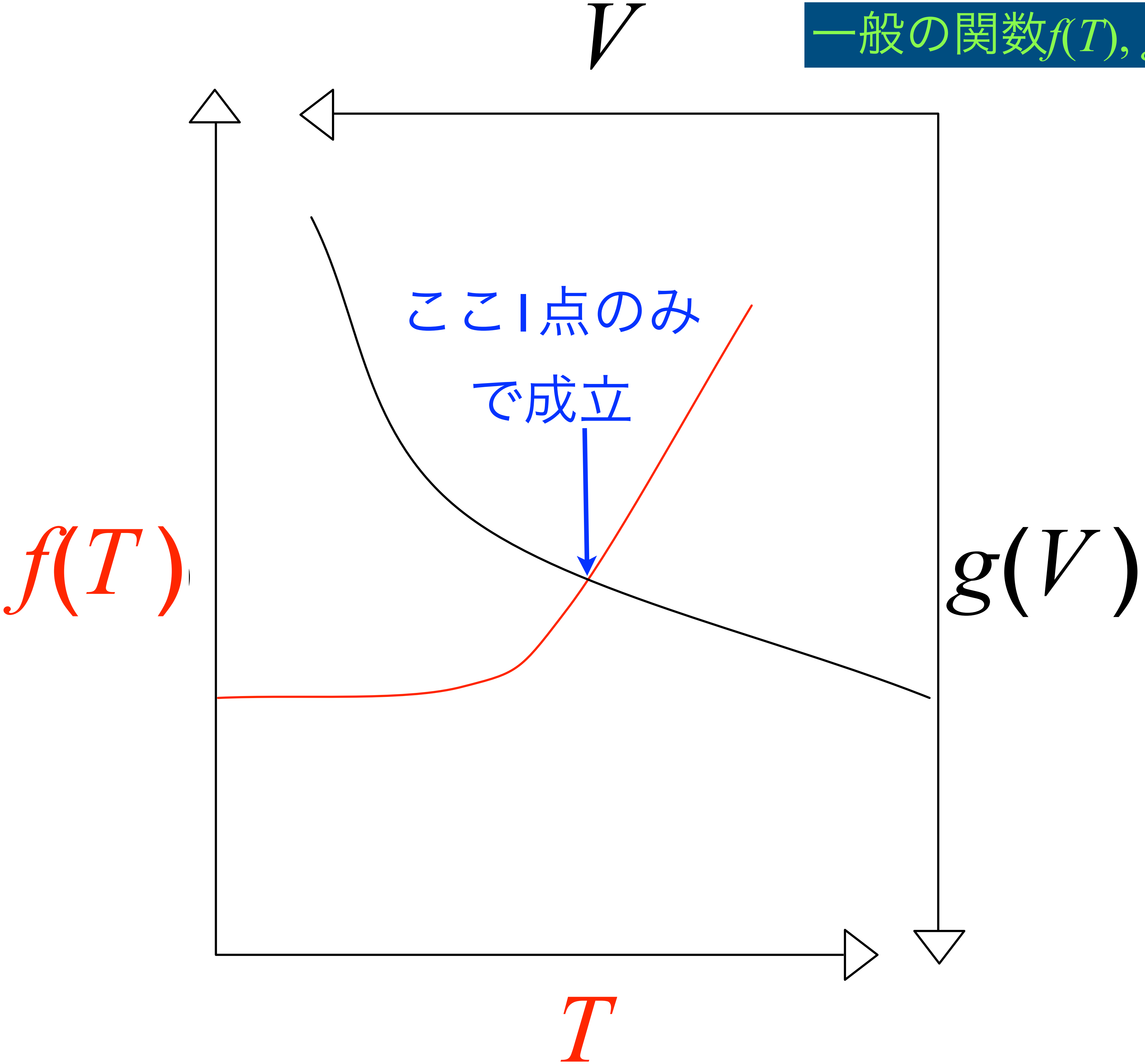
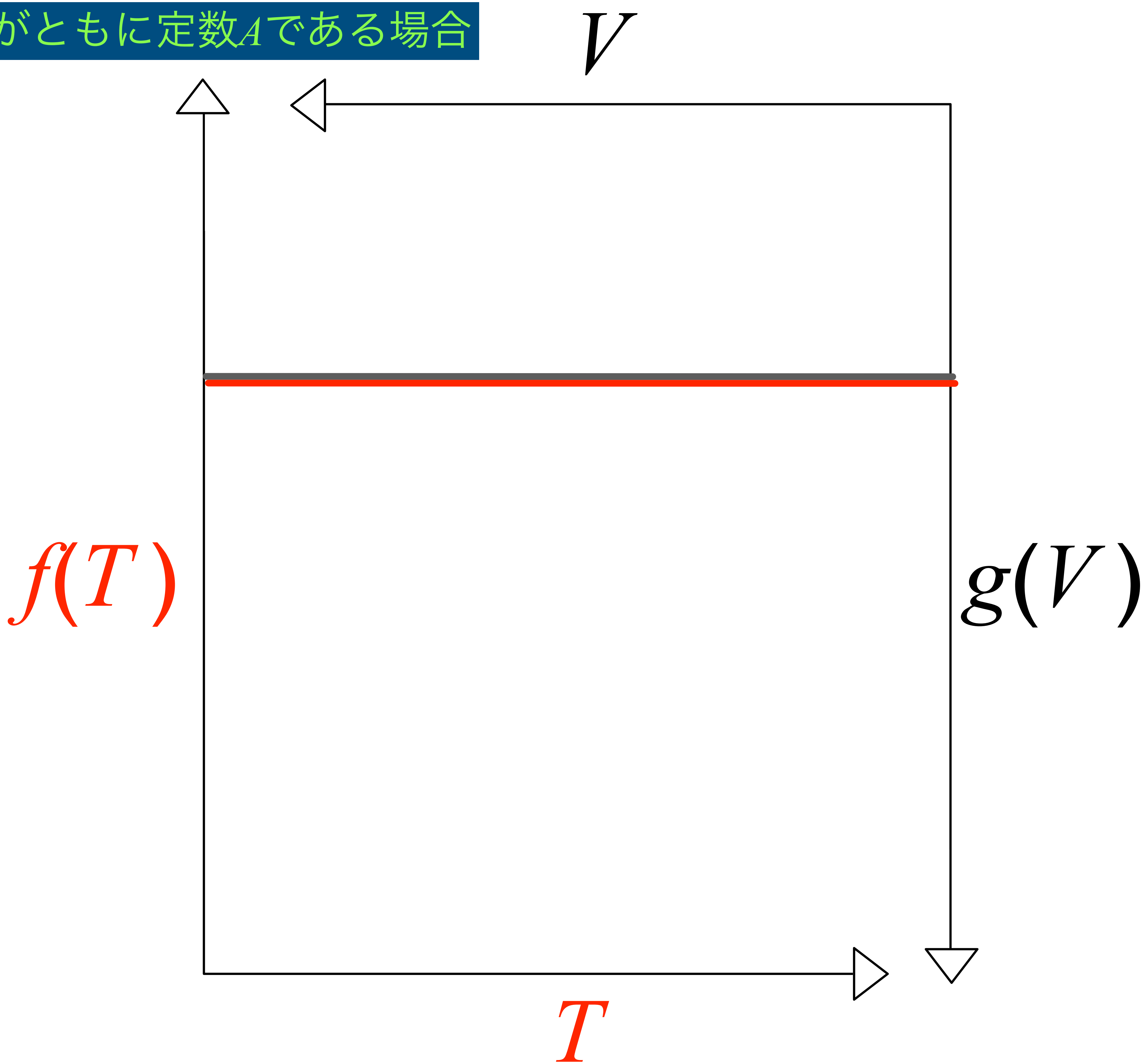


$f(T), g(V)$がともに定数$A$である場合



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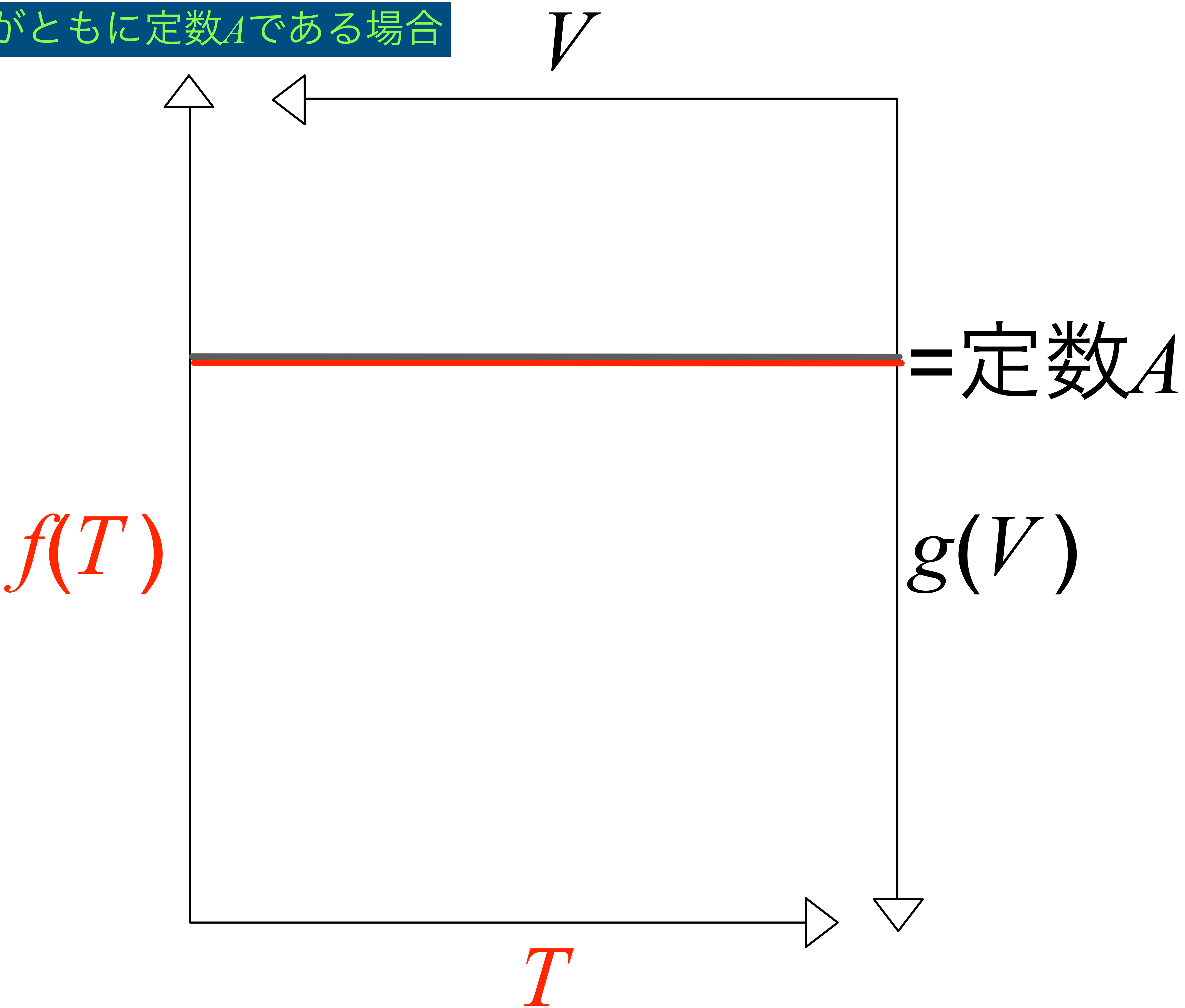

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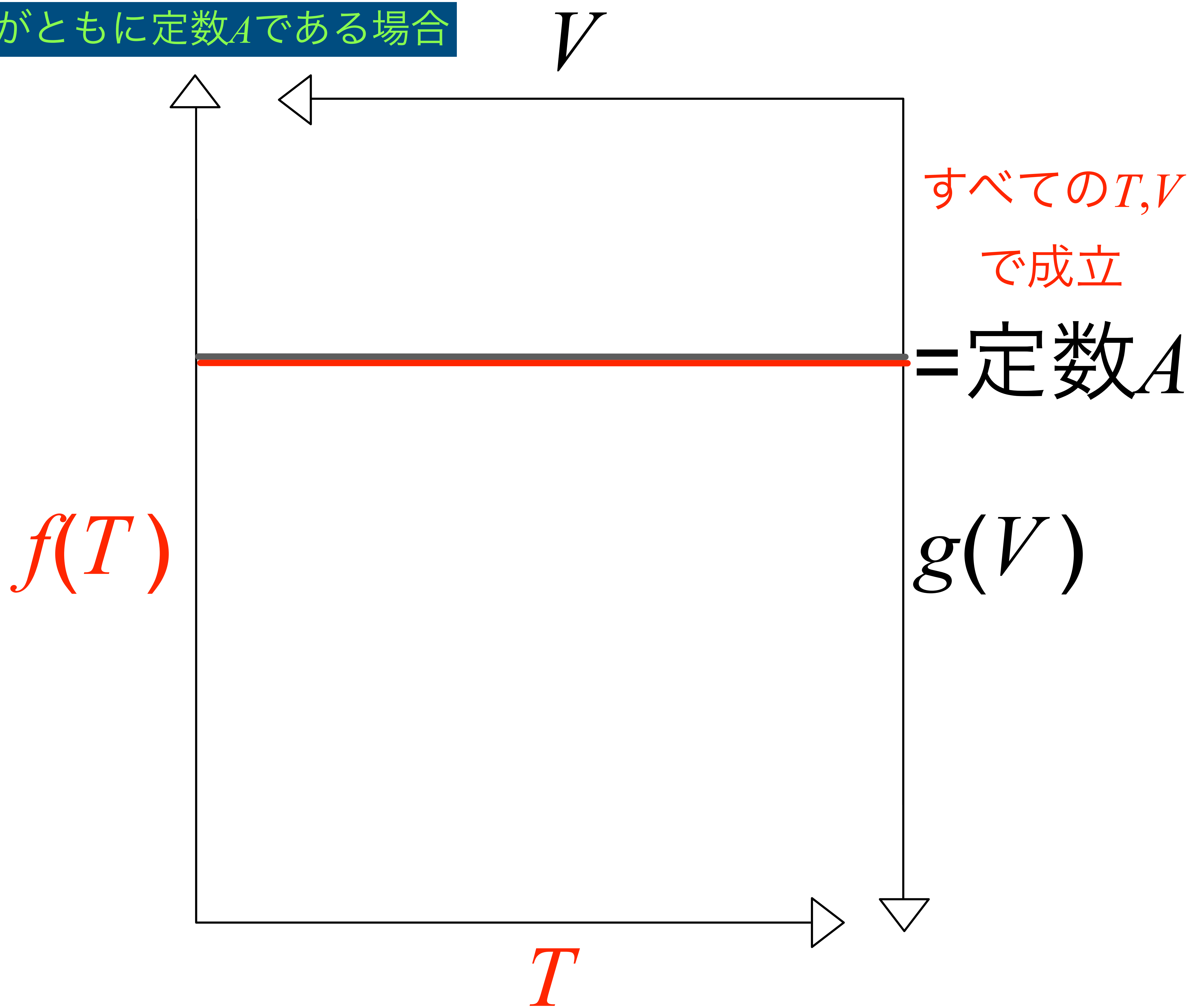


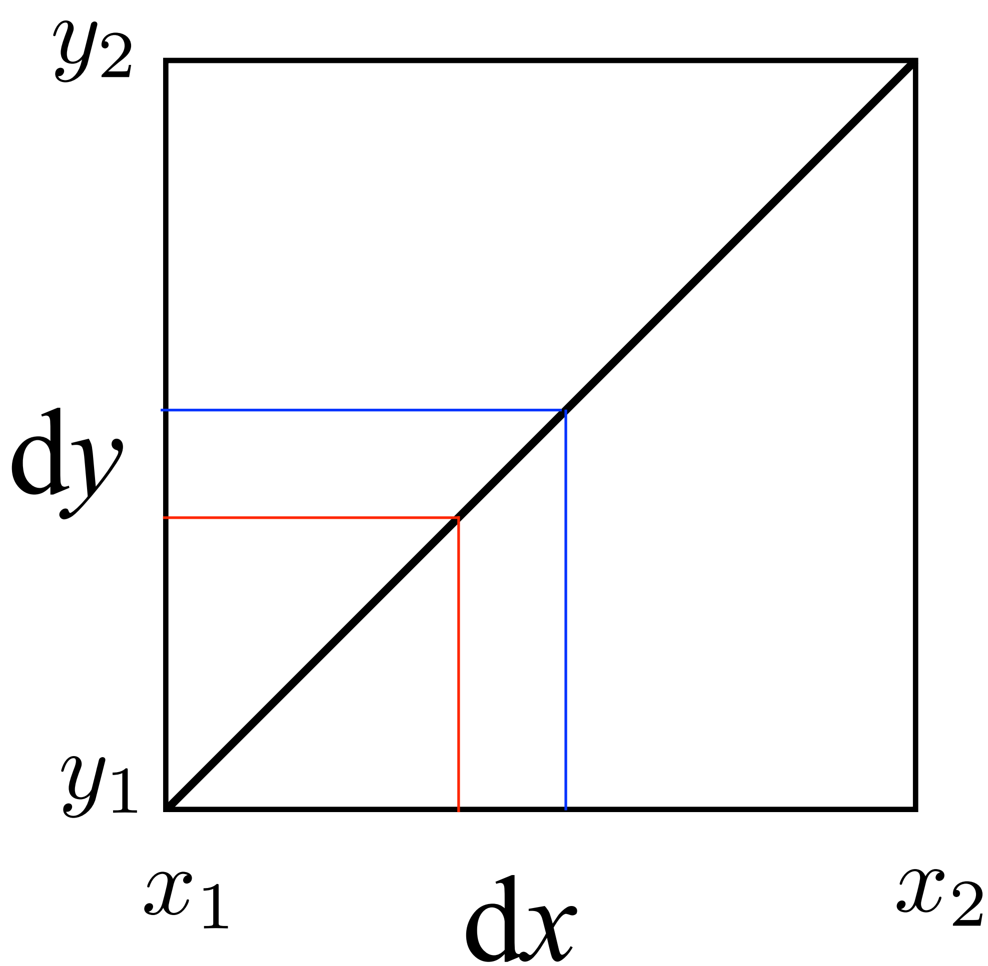
$$c\frac{\mathrm{d}T}{T} = -\frac{\mathrm{d}V}{V} = A\text{(定数)}$$

$$\frac{\mathrm{d}\ln T}{\mathrm{d}T} = \frac{1}{T},\ \mathrm{d}(\ln T) = \frac{\mathrm{d}T}{T}$$

$$\mathrm{d}(c\ln T) = \mathrm{d}(-\ln V) = A\text{(定数)}$$

$\mathrm{d}(c\ln T) = \mathrm{d}c \times \ln T + c\mathrm{d}(\ln T) = 0 \times \ln T + c\mathrm{d}(\ln T)$

$\mathrm{d}(-\ln V) = \mathrm{d}(-1) \times \ln V - \mathrm{d}(\ln V) = 0 \times \ln V - \mathrm{d}(\ln V)$




$$\mathrm{d}x = \mathrm{d}(c \ln T),\ \ \mathrm{d}y = \mathrm{d}(-\ln V)$$

とおく

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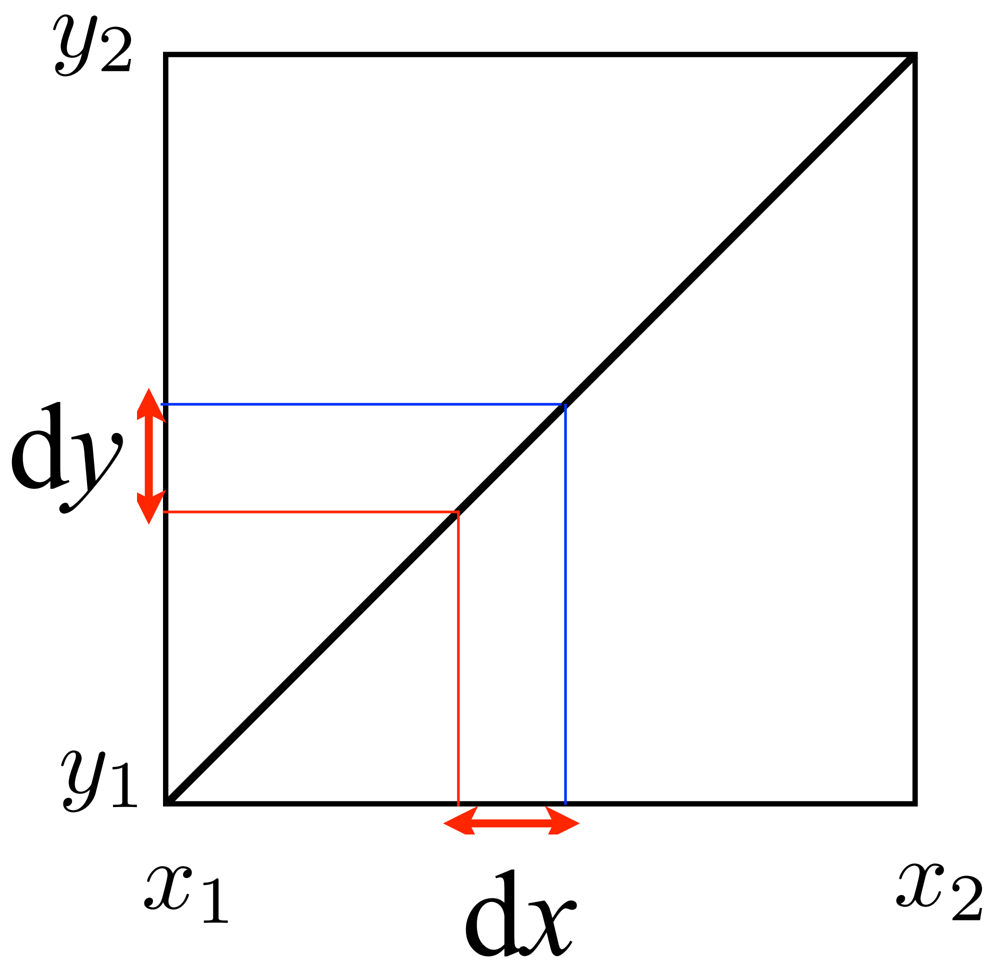
となる。これを，以下のように図であらわす。

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$y_2$

短冊状に足してゆくと

$\mathrm{d}y$

$$x_2 - x_1 = y_2 - y_1$$

$y_1$

$x_1$ $\mathrm{d}x$ $x_2$

$$c \ln T_2 - c \ln T_1 = -\ln V_2 - (-\ln V_1)$$

$$c \ln \frac{T_2}{T_1} = \ln \frac{V_1}{V_2}$$

$$\ln \left(\frac{T_2}{T_1}\right)^c = \ln \frac{V_1}{V_2}$$

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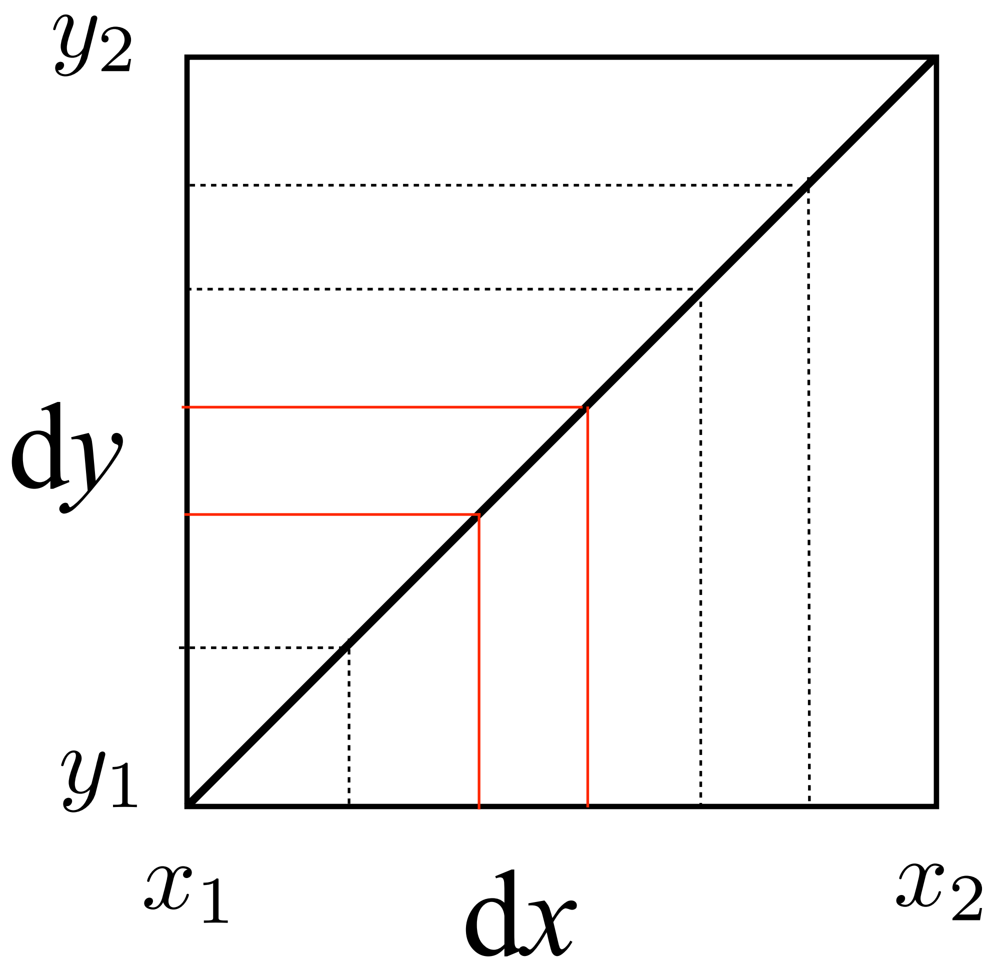
$$V_1 T_1^c = V_2 T_2^c$$

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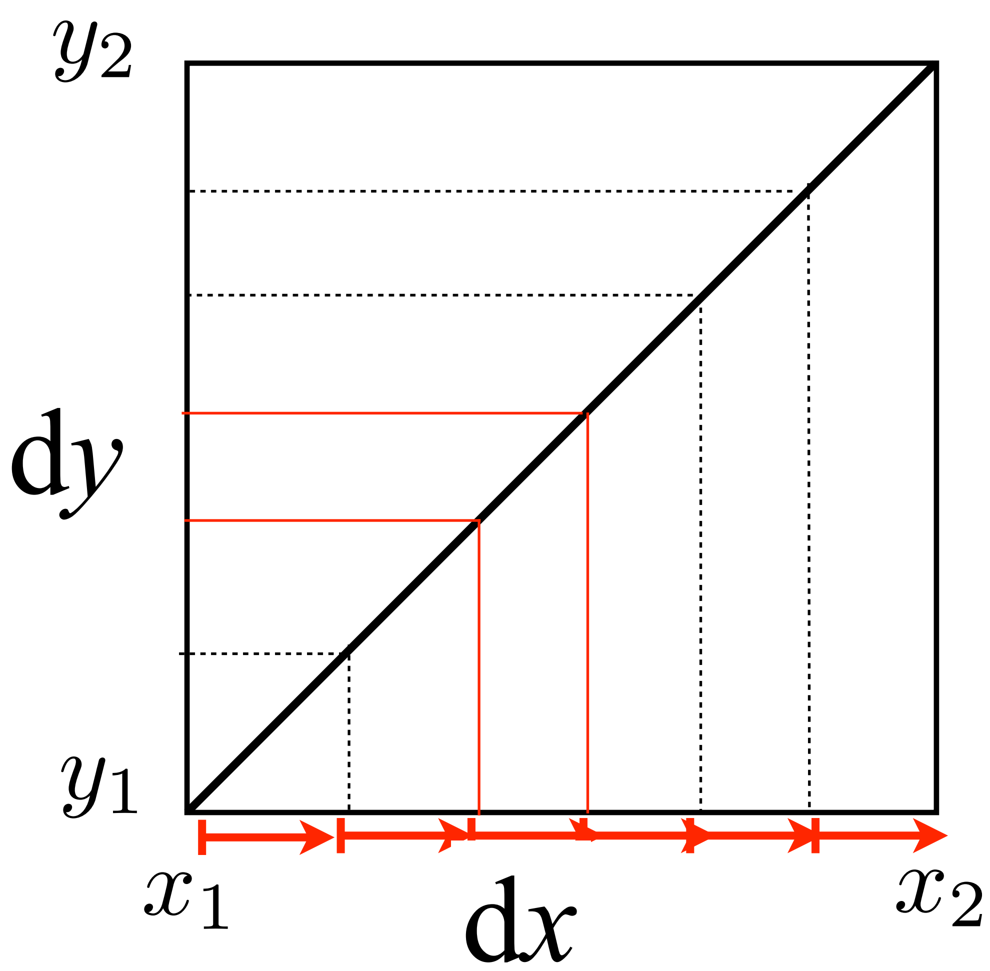


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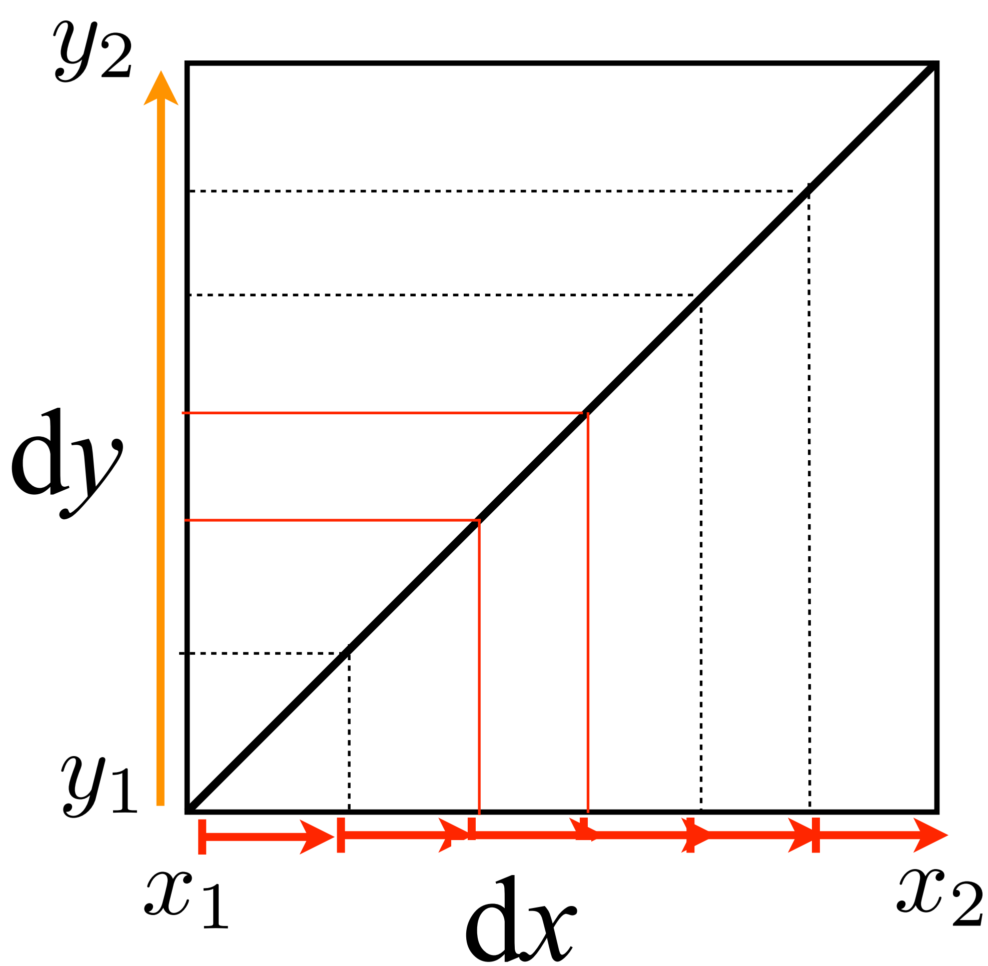


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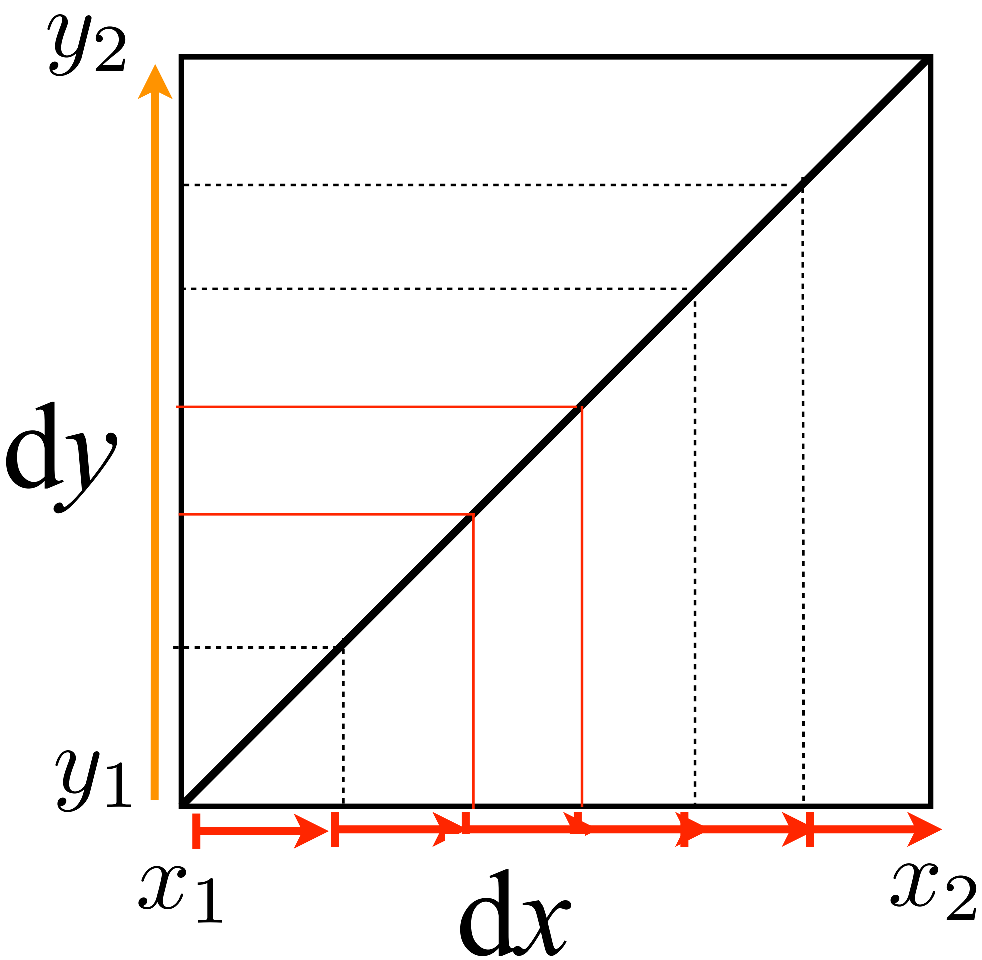


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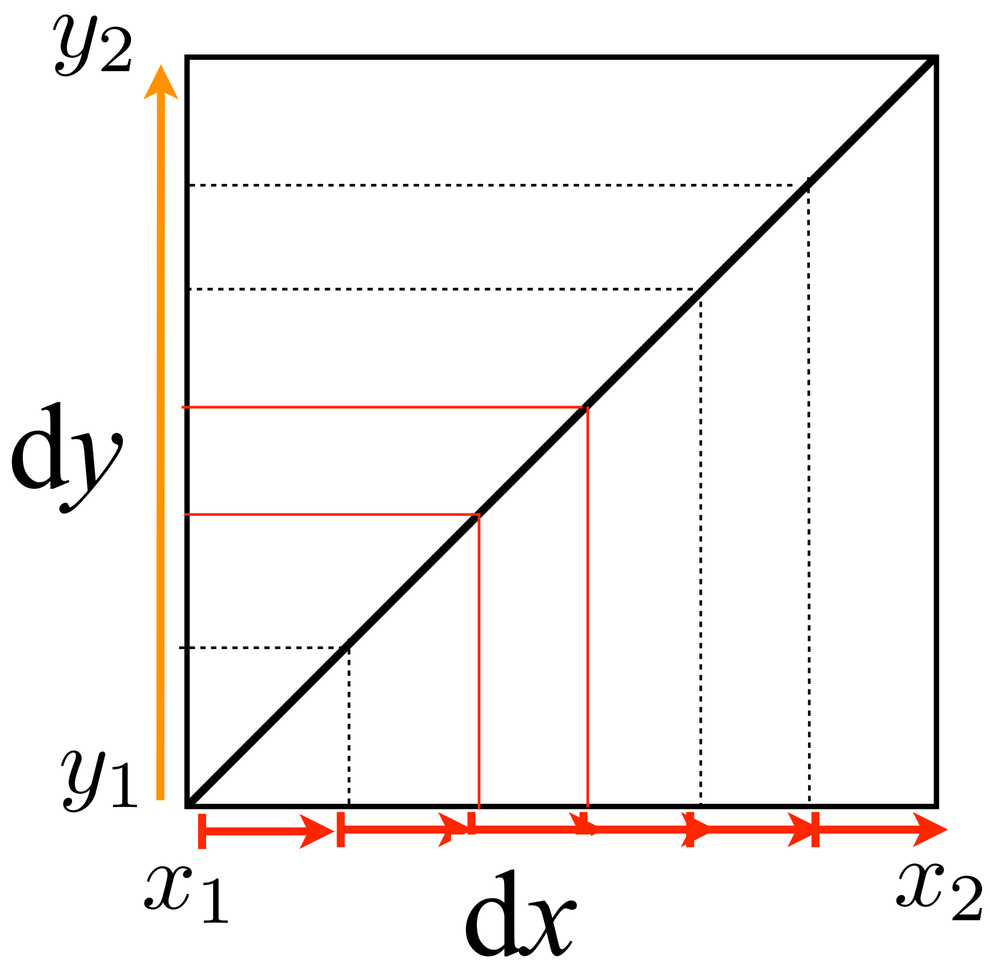


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# 別解（こちらがより一般的）

$T$: $T_1 \to T_2$

$V$: $V_1 \to V_2$

$c\,\mathrm{d}T / T = -\,\mathrm{d}V / V$　をこの範囲で別々に積分する

$$c\int_{T_1}^{T_2} \frac{1}{T}\mathrm{d}T = -\int_{V_1}^{V_2} \frac{1}{V}\mathrm{d}V$$

$$c\int_{T_1}^{T_2} \frac{\mathrm{d}\ln T}{\mathrm{d}T}\mathrm{d}T = -\int_{V_1}^{V_2} \frac{\mathrm{d}\ln V}{\mathrm{d}V}\mathrm{d}V$$

$$c\left[\ln T\right]_{T_1}^{T_2} = -\left[\ln V\right]_{V_1}^{V_2}$$

$$c(\ln T_2 - \ln T_1) = -(\ln V_2 - \ln V_1)$$

$$c\ln\frac{T_2}{T_1} = \ln\frac{V_1}{V_2},\ \ln\left(\frac{T_2}{T_1}\right)^c = \frac{V_1}{V_2}$$

$$\left(\frac{T_2}{T_1}\right)^c = \ln\frac{V_1}{V_2},\ V_1T_1^c = V_2T_2^c$$

## Adiabatic changes - Poisson equations

断熱変化　　　ポアソンの関係式

$$V_1 T_1^c = V_2 T_2^c,\ VT^c = \mathrm{const},\ (VT^c)^{1/c} = V^{1/c}T = \mathrm{const}$$

$$PV = nRT, T = PV/(nR),\ V^{1/c}PV = \mathrm{const}$$

$$PV^{\frac{c+1}{c}} = \mathrm{const}$$

$$c = \frac{C_V}{nR},\ \frac{c+1}{c} = \frac{C_V + nR}{C_V} = \frac{C_P}{C_V} \equiv \gamma$$

比熱比（heat capacity ratio)

$$PV^\gamma = \mathrm{const}$$

# 可逆的断熱膨張：仕事

$T$: $T_\mathrm{H} \longrightarrow T_\mathrm{L}$ $\qquad V_2 < V_3$

$V$: $V_2 \longrightarrow V_3$ $\qquad P_2 V_2 = nRT_\mathrm{H}$

$P$: $P_2 \longrightarrow P_3$ $\qquad P_3 V_3 = nRT_\mathrm{L}$

$$
\begin{aligned}
PV^\gamma &= \mathrm{const} = P_2 V_2^\gamma = P_3 V_3^\gamma \\
\Delta W &= -\int_{V_2}^{V_3} P\mathrm{d}V = -P_2 V_2^\gamma \int_{V_2}^{V_3} V^{-\gamma}\mathrm{d}V \\
&= -P_2 V_2^\gamma \left[\frac{1}{-\gamma+1} V^{-\gamma+1}\right]_{V_2}^{V_3} = -P_2 V_2^\gamma \frac{1}{-\gamma+1}\left(V_3^{-\gamma+1} - V_2^{-\gamma+1}\right) \\
&= \frac{1}{-\gamma+1}\left(-P_3 V_3^\gamma V_3^{-\gamma+1} + P_2 V_2^\gamma V_2^{-\gamma+1}\right) \\
&= \frac{1}{-\gamma+1}\left(-P_3 V_3 + P_2 V_2\right) = \frac{nR}{-\gamma+1}\left(-T_\mathrm{L} + T_\mathrm{H}\right) \\
&= \frac{nR}{-C_P/C_V + 1}\left(-T_\mathrm{L} + T_\mathrm{H}\right) = C_V \frac{nR}{-(C_P - C_V)}\left(-T_\mathrm{L} + T_\mathrm{H}\right) \\
&= -C_V \frac{C_P - C_V}{C_P - C_V}\left(-T_\mathrm{L} + T_\mathrm{H}\right) = C_V\left(T_\mathrm{L} - T_\mathrm{H}\right) < 0
\end{aligned}
$$