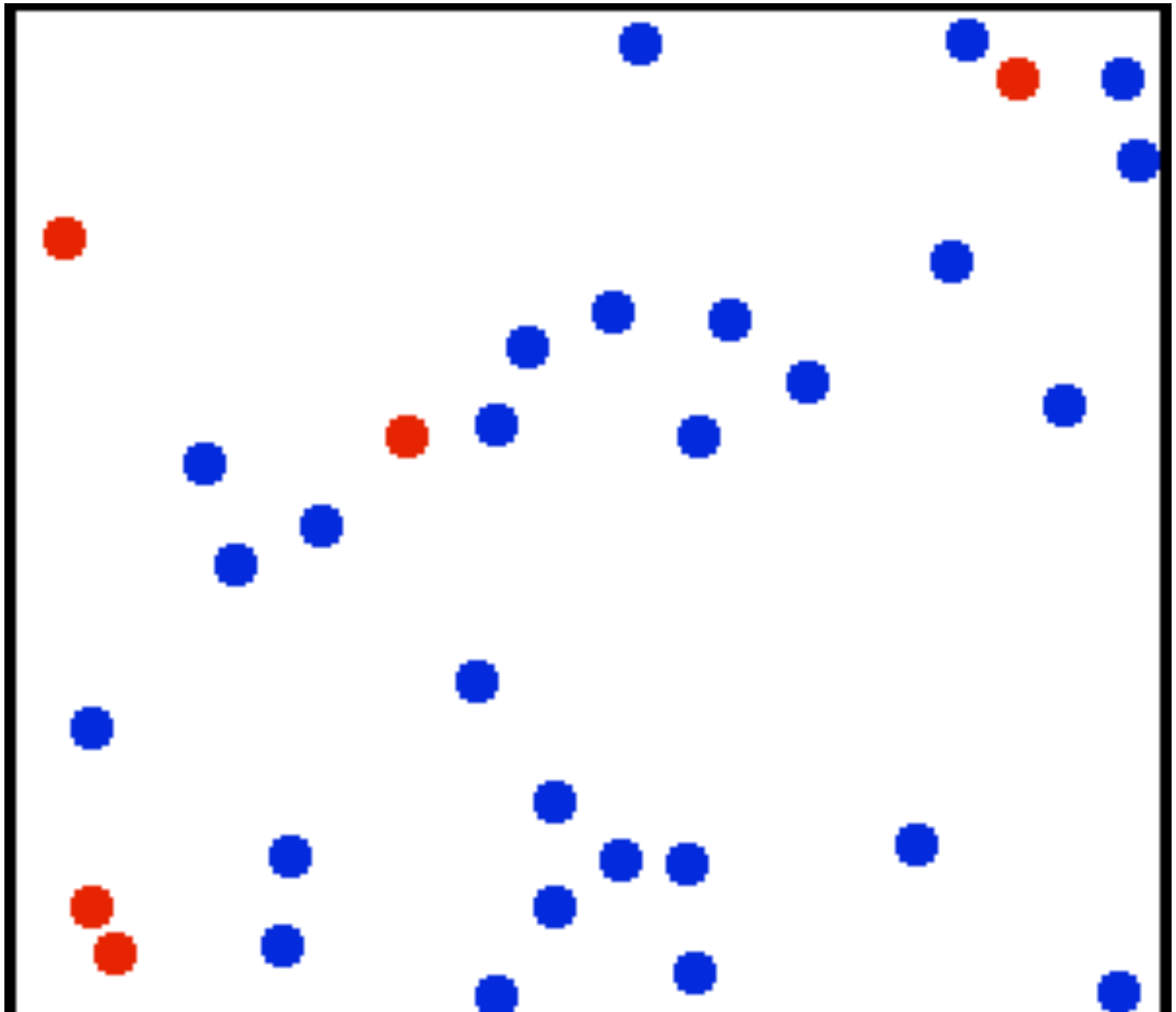
分子運動論

あるる温度の時にどのような速度で飛び回っているのか？



理想気体：大きさなし，相互作用無し

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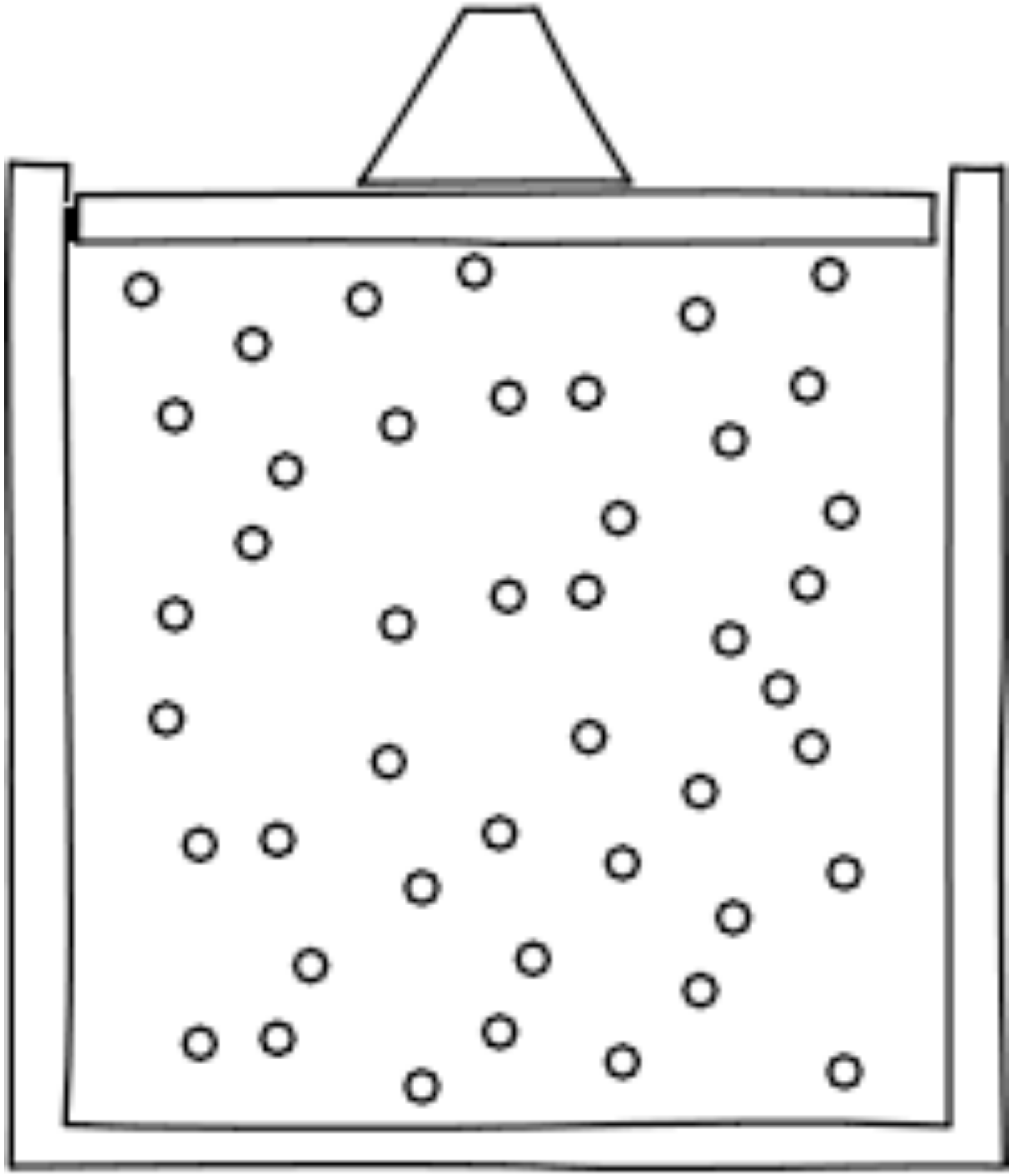
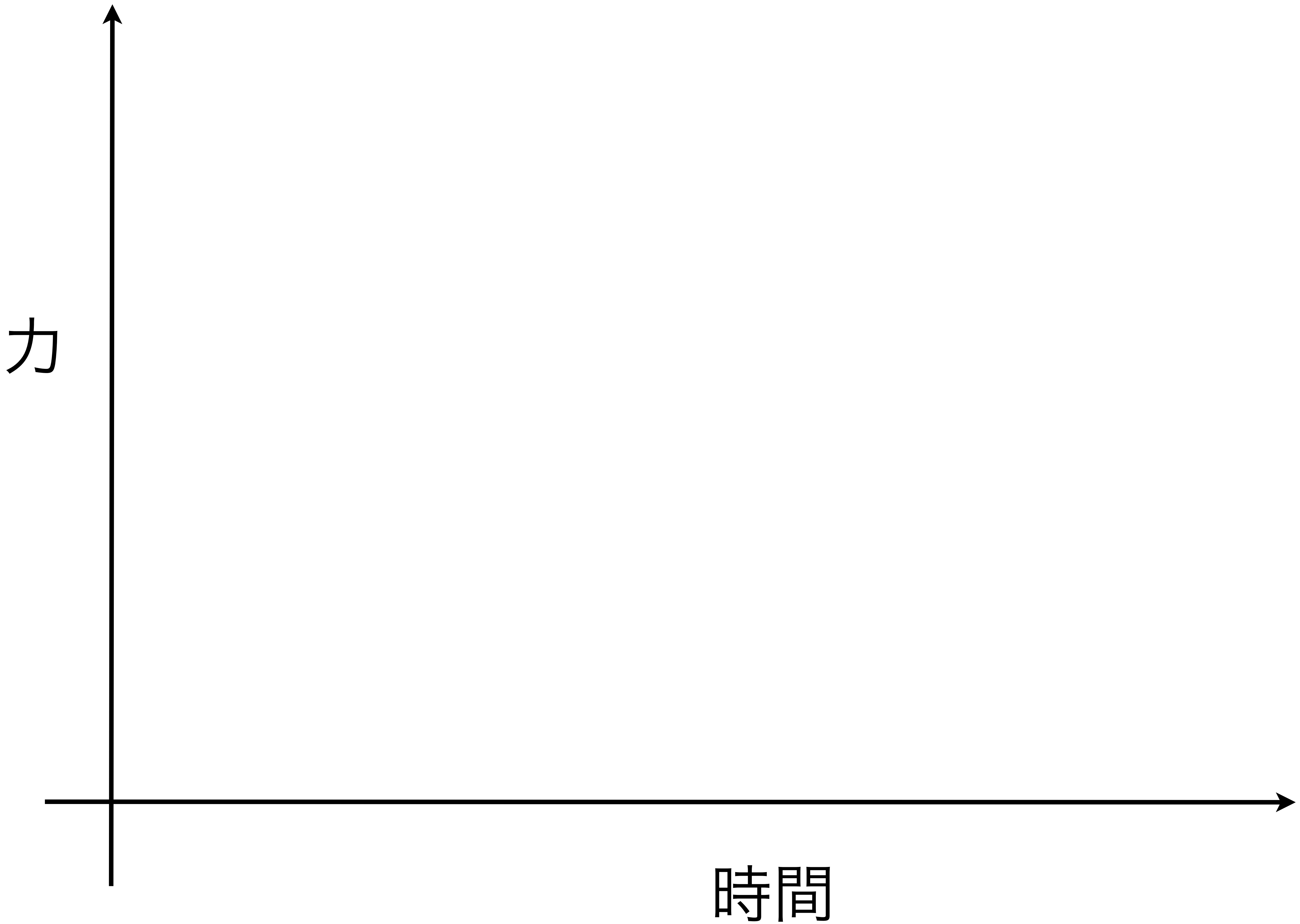
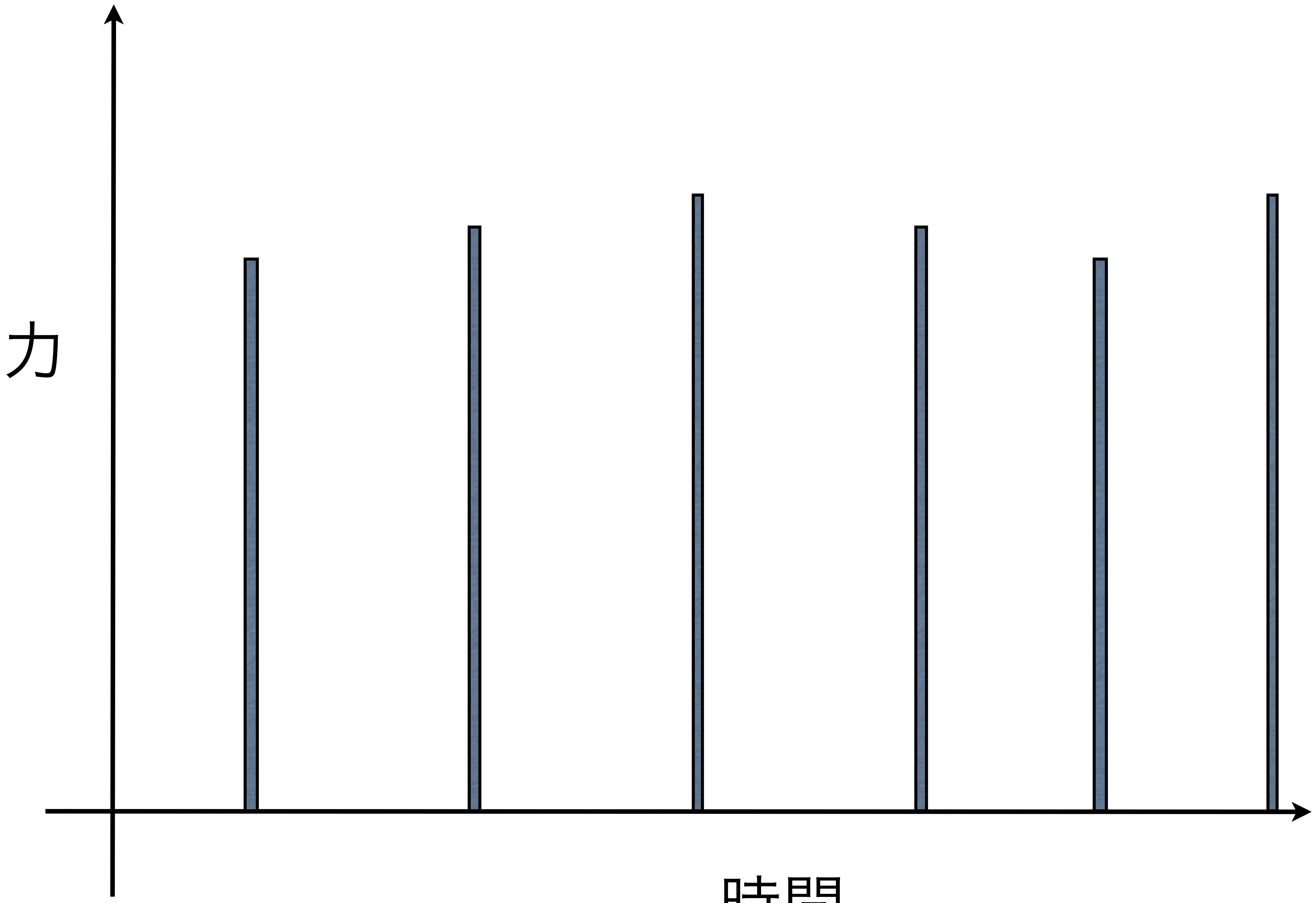
内部エネルギー ＝ 運動エネルギーの総和

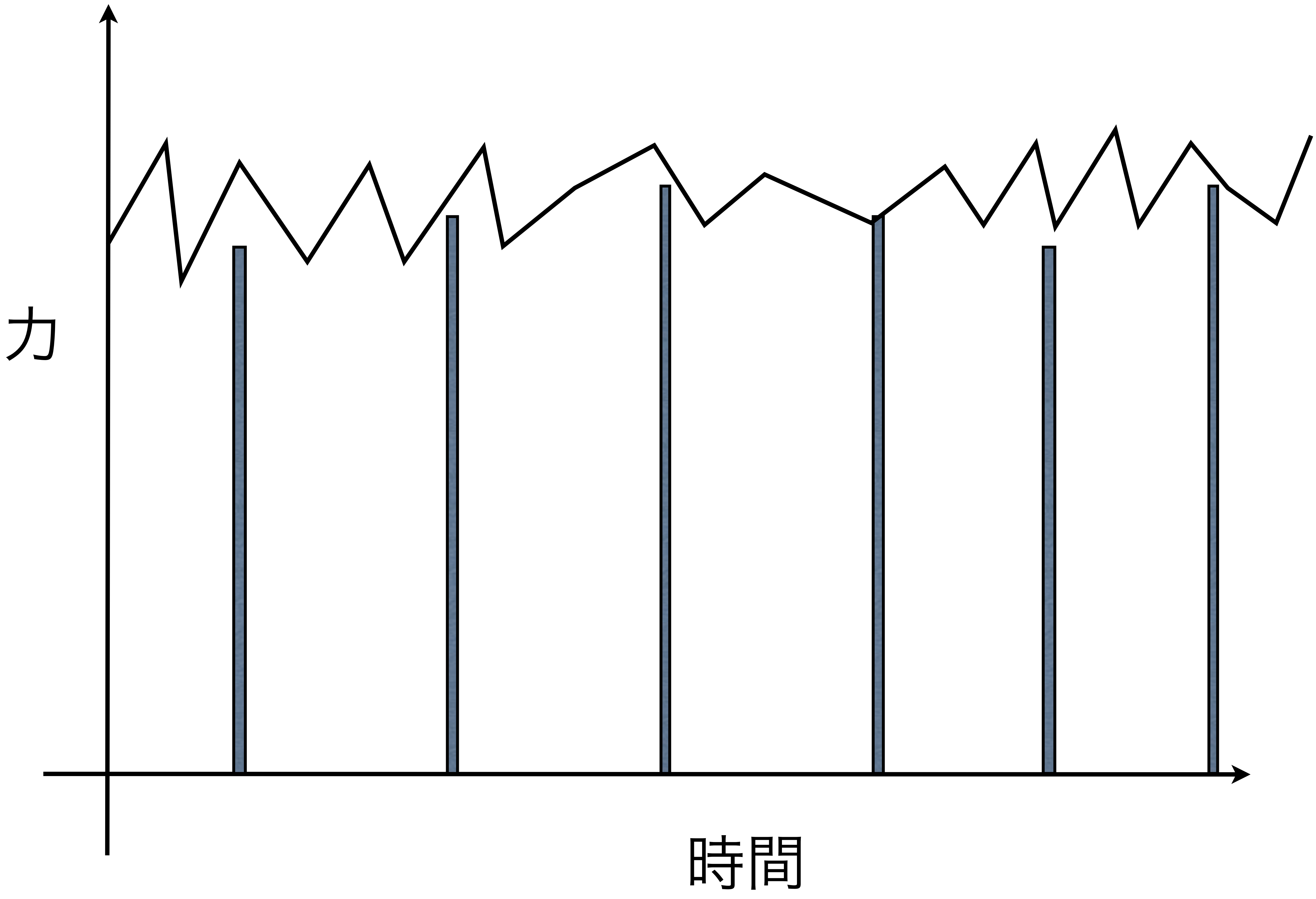
→　温度のみの関数

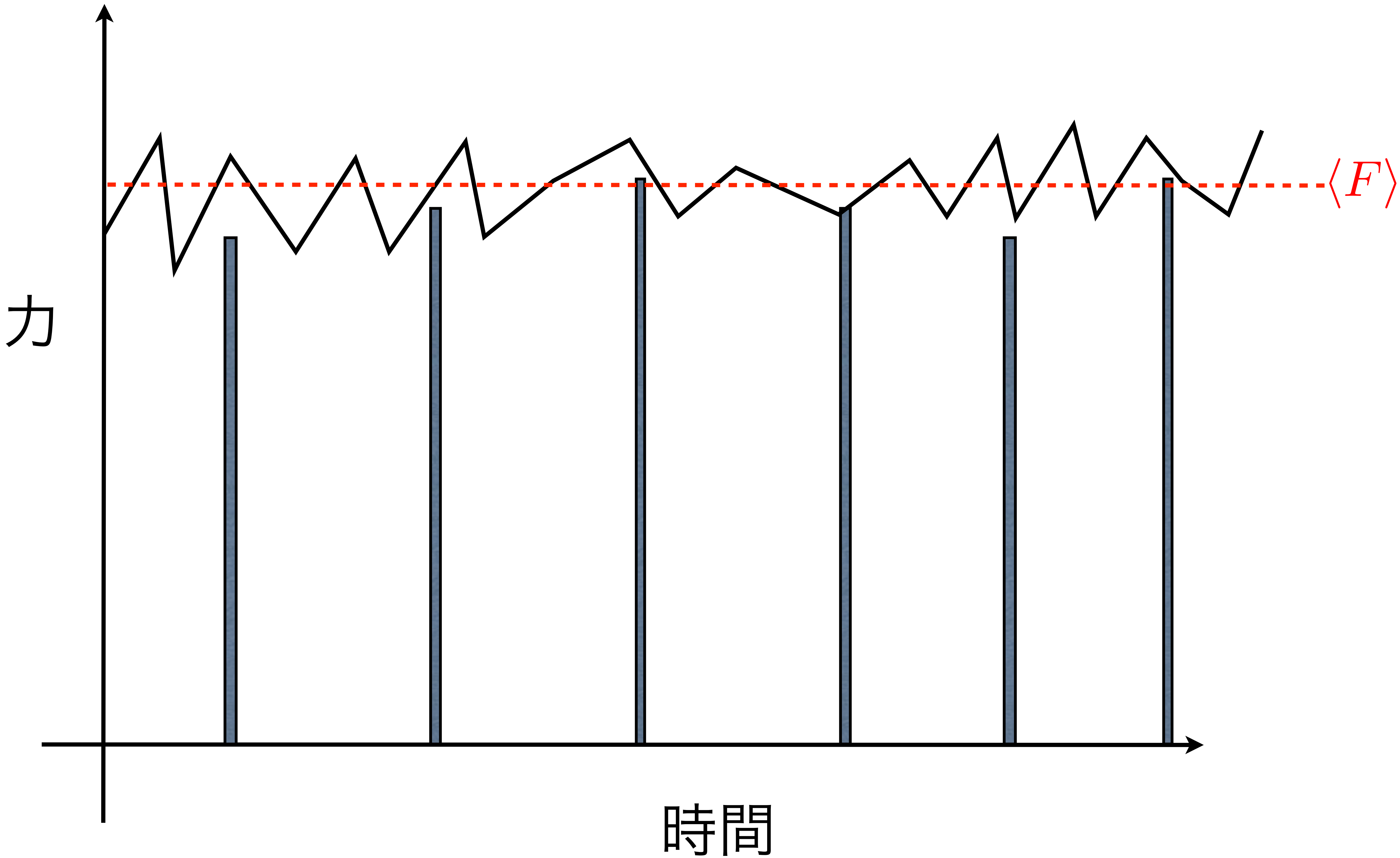


圧力
すなわち
理想気体の
状態方程式
を
導こう


力
時間


力
時間


力
時間


力
時間
⟨F⟩

理想気体の状態方程式と力学

$$F = ma = m\frac{dv}{dt}$$

$$\underbrace{\int_{t_1}^{t_2} F dt}_{\text{力積}} = \int_{t_1}^{t_2} m\frac{dv}{dt}dt = \underbrace{mv(t_2) - mv(t_1)}_{\text{運動量変化}}$$

今度は時間で積分

$$\int_{t_1}^{t_2} F dt \simeq \underbrace{\langle F \rangle}_{\text{力の時間平均(定数)}} \underbrace{\Delta t}_{=t_2-t_1} = m[v(t_2) - v(t_1)]$$

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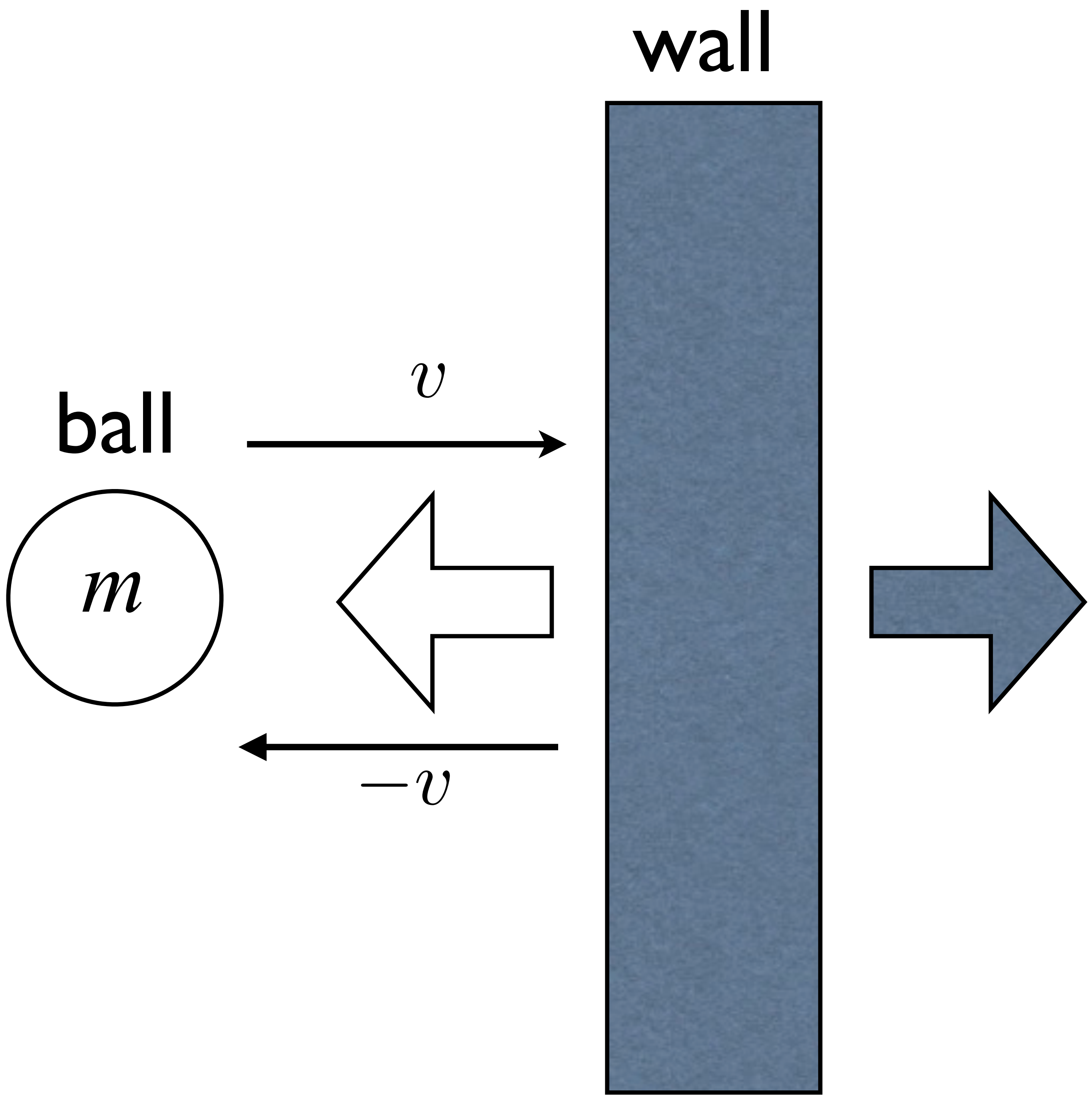
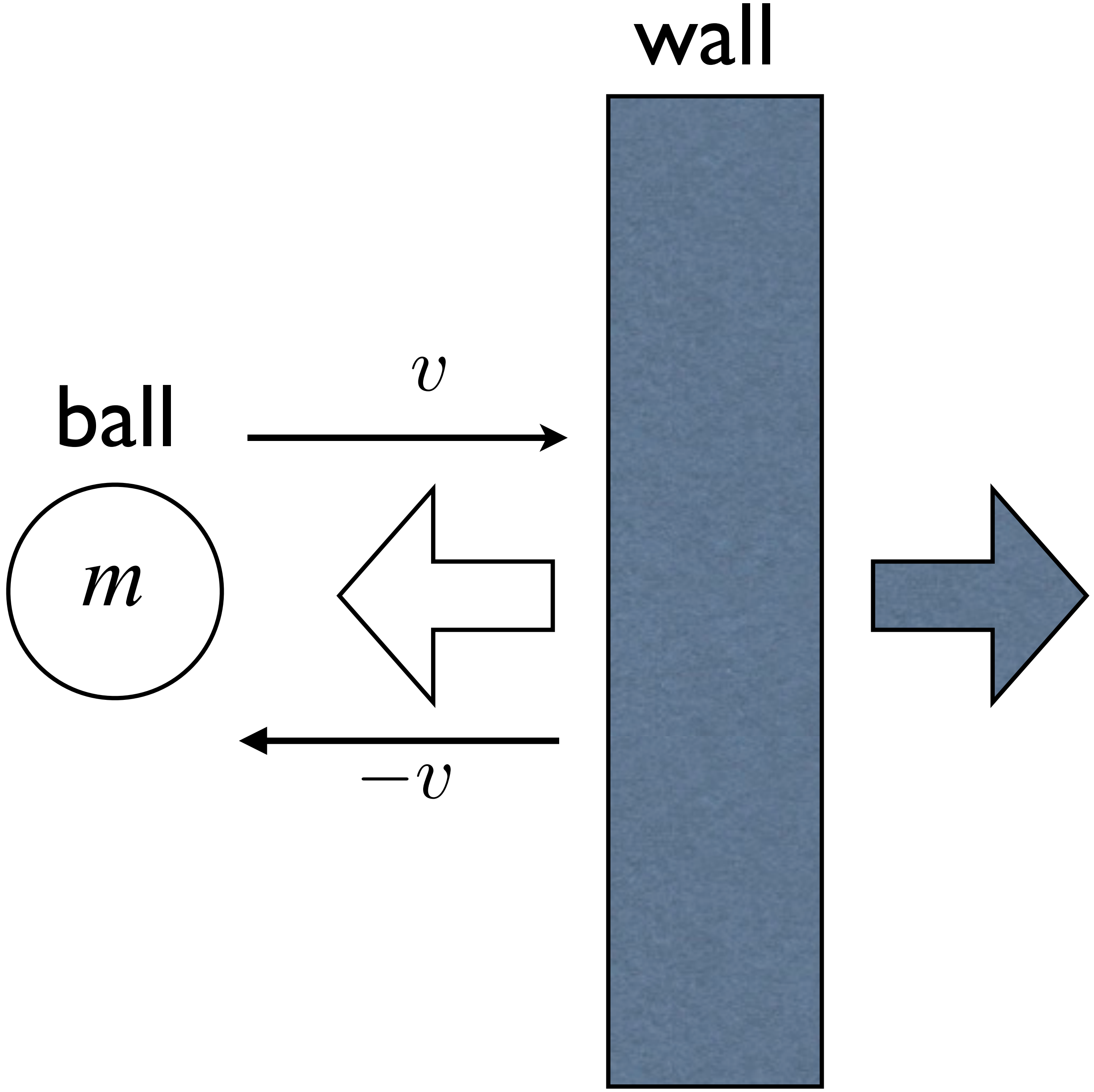


after before

$$\langle F_{\rm ball}\rangle \Delta t = m(-v - v) = -2mv$$

$$\langle F_{\rm ball}\rangle \Delta t = -\langle F_{\rm wall}\rangle \Delta t$$:作用・反作用

$$\langle F_{\rm wall}\rangle \Delta t = 2mv$$



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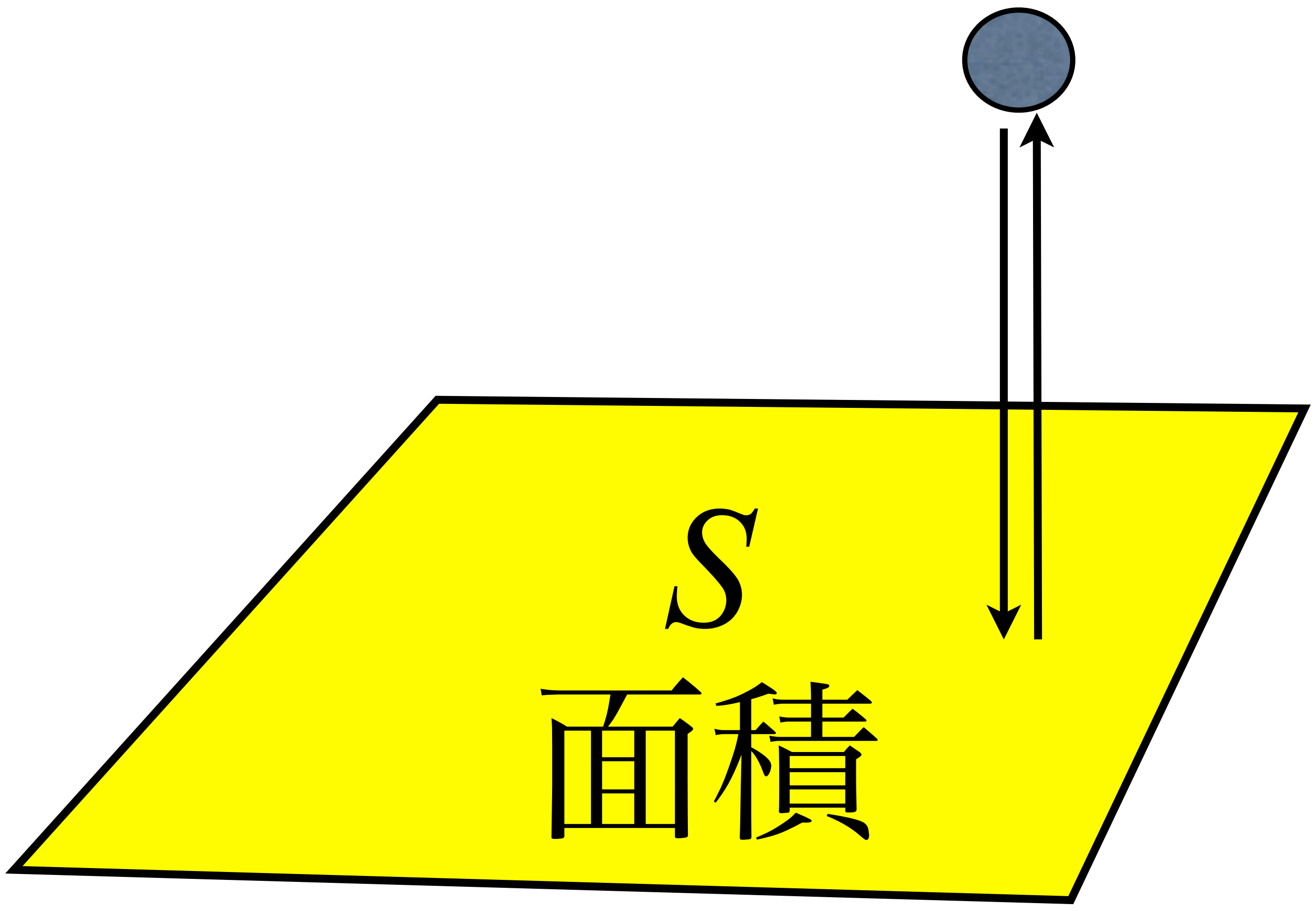
$$< F_{\text{wall}} > = PS$$

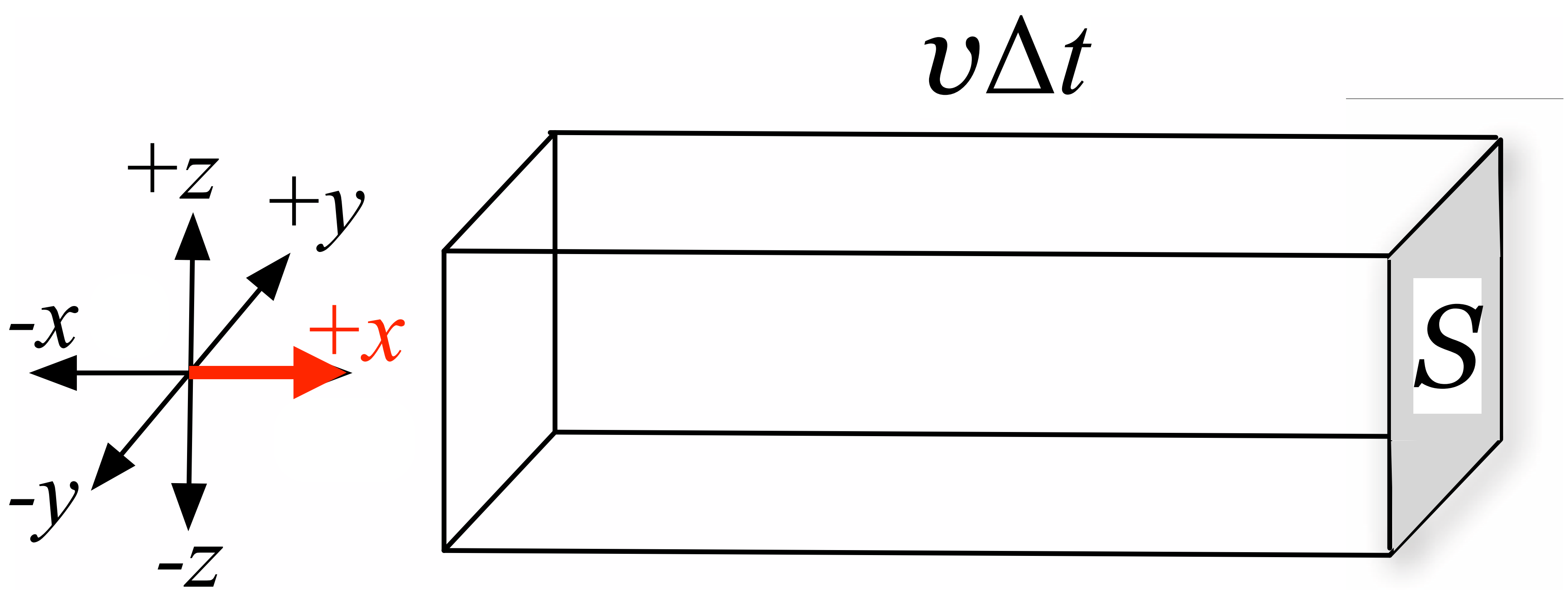


気体は，体積V [m^3],　分子数Nで存在するとする

気体は速度vで$\pm x, \pm y, \pm z$のみに運動

以下の直方体内にある分子の1/6は全てΔt秒間に気体が壁を叩く。Δt秒間に気体が壁を叩く分子数は

$$\frac{1}{6}(Sv\Delta t)\frac{N}{V}$$



$$\langle F_{\text{wall}} \rangle = PS = \frac{2mv}{\Delta t}\frac{1}{6}(Sv\Delta t)\frac{N}{V}$$

$$P = \frac{1}{3}\frac{N}{V}mv^2$$

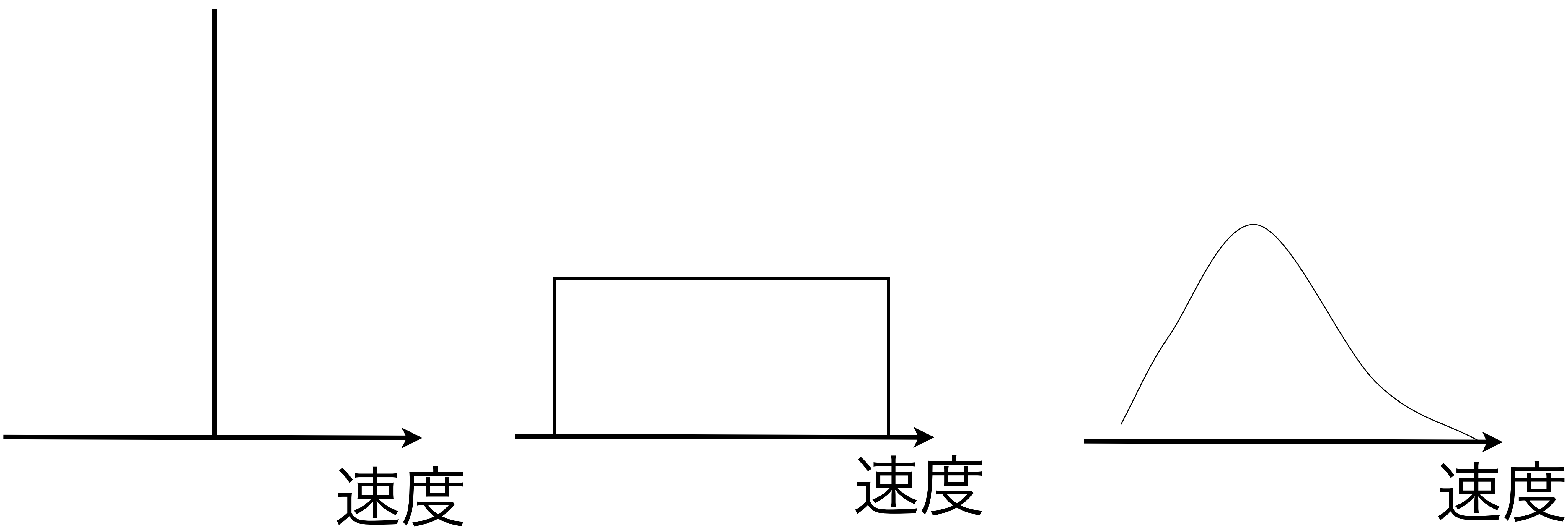
$$PV = \frac{2}{3}N(\frac{1}{2}mv^2) = \frac{2}{3}E_{\text{kinetic}}$$

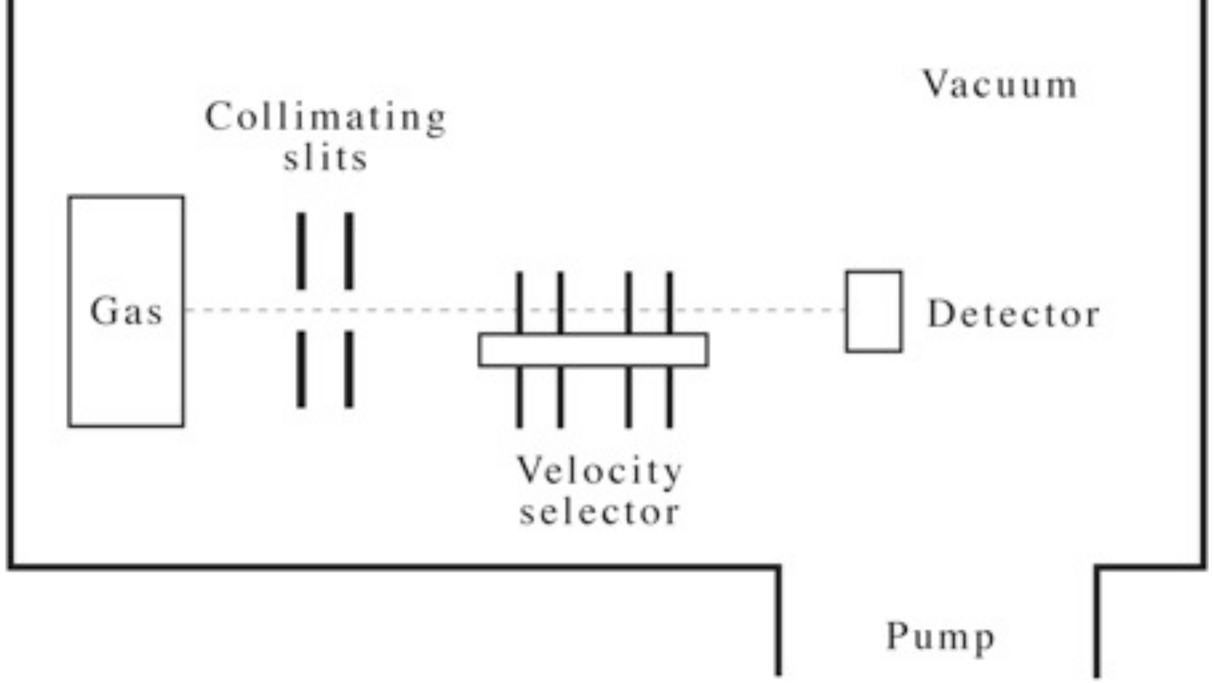
$$E_{\text{kinetic}} = \frac{3}{2}nRT \quad \text{(from statistical mechanics)}$$

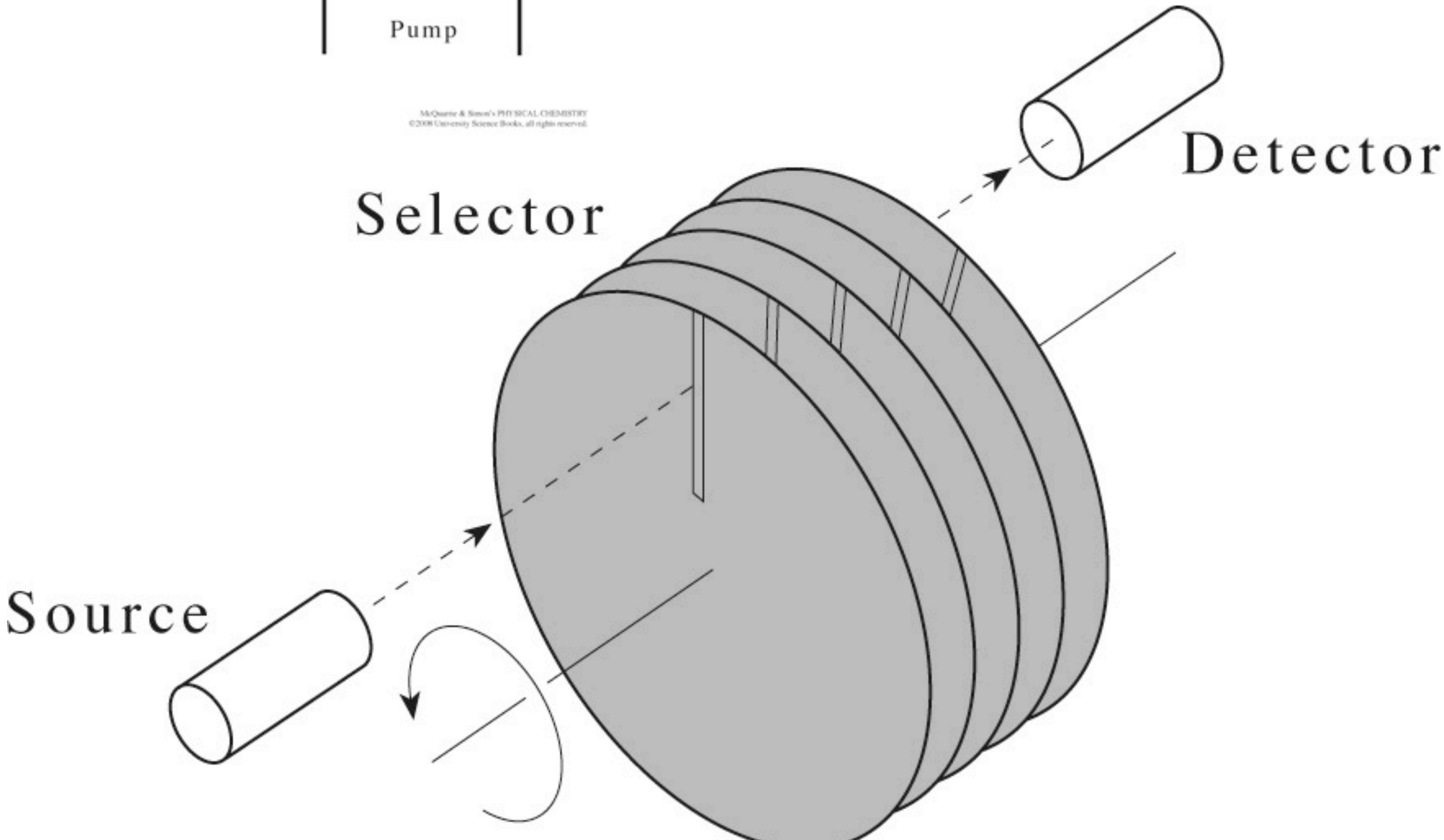
$$PV = \frac{2}{3}\frac{3}{2}nRT = nRT$$

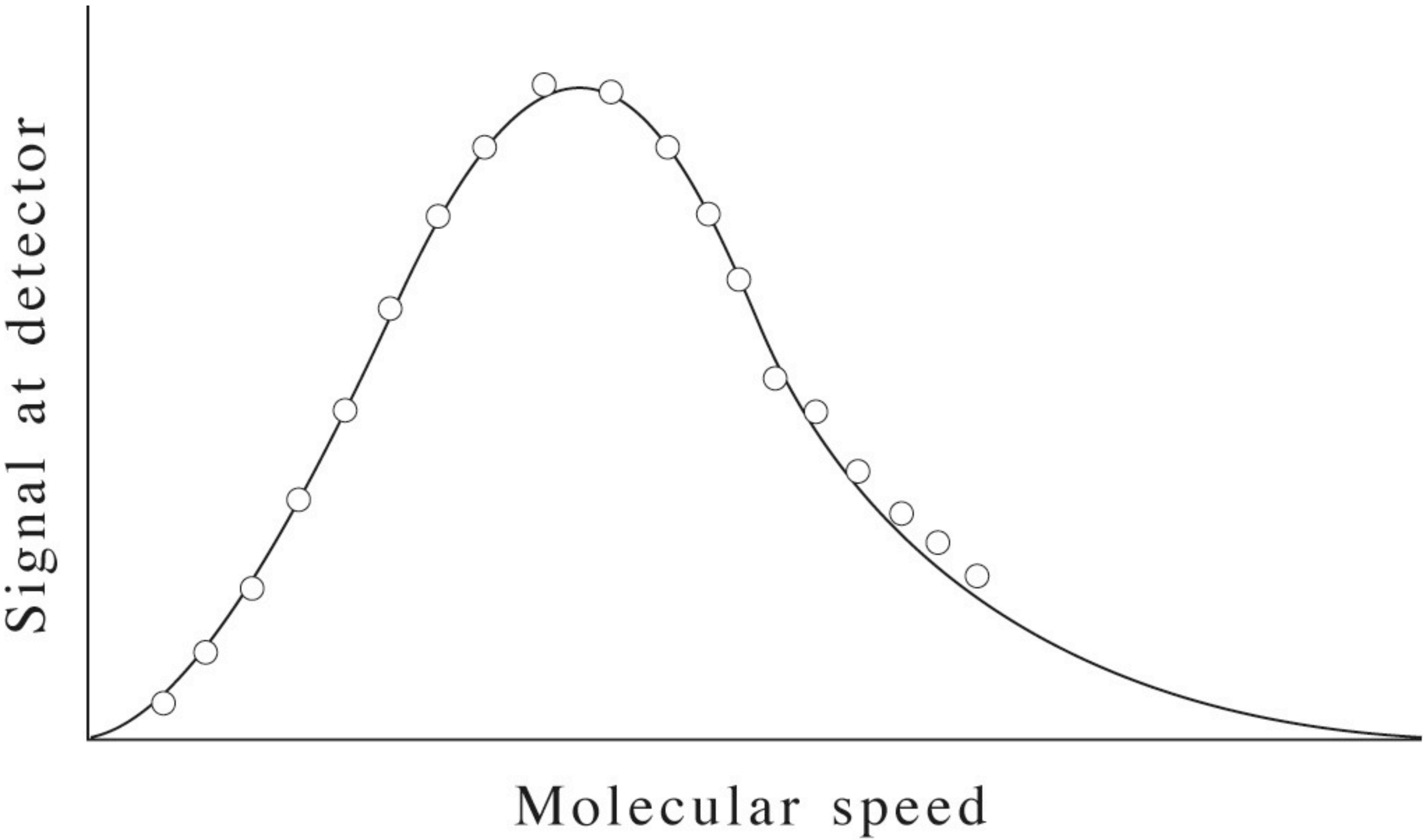
速度は分布を持つ？

実験結果は？




Vacuum
Collimating slits
Gas
Detector
Velocity selector
Pump


Detector
Selector
Source


Signal at detector
Molecular speed

PHYSICAL REVIEW VOLUME 99, NUMBER 4 AUGUST 15, 1955

Velocity Distributions in Potassium and Thallium Atomic Beams*†

R. C. MILLER‡ AND P. KUSCH
Columbia University, New York, New York

(Received February 23, 1955)

A high-resolution, high-intensity, spiral velocity selector has been designed for the study of the velocity distributions of the components of atomic and molecular beams. It has been found possible to design oven slits which closely approximate the ideal aperture of kinetic theory. An analysis has been made of the velocity distributions in beams of potassium and thallium over a range of velocity from 0.3 to 2.5 times the most probable velocity in the oven. The agreement between the observed distribution and that deduced, on the basis of the assumptions that the distribution in the oven is Maxwellian and that the aperture is ideal, is very good.

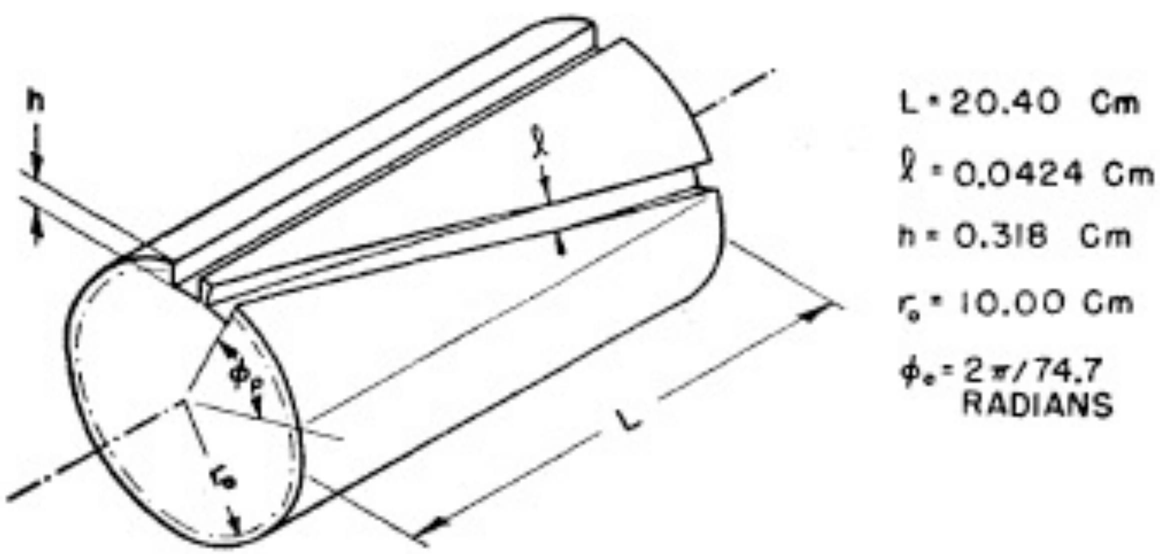


FIG. 1. Schematic diagram of velocity selector.

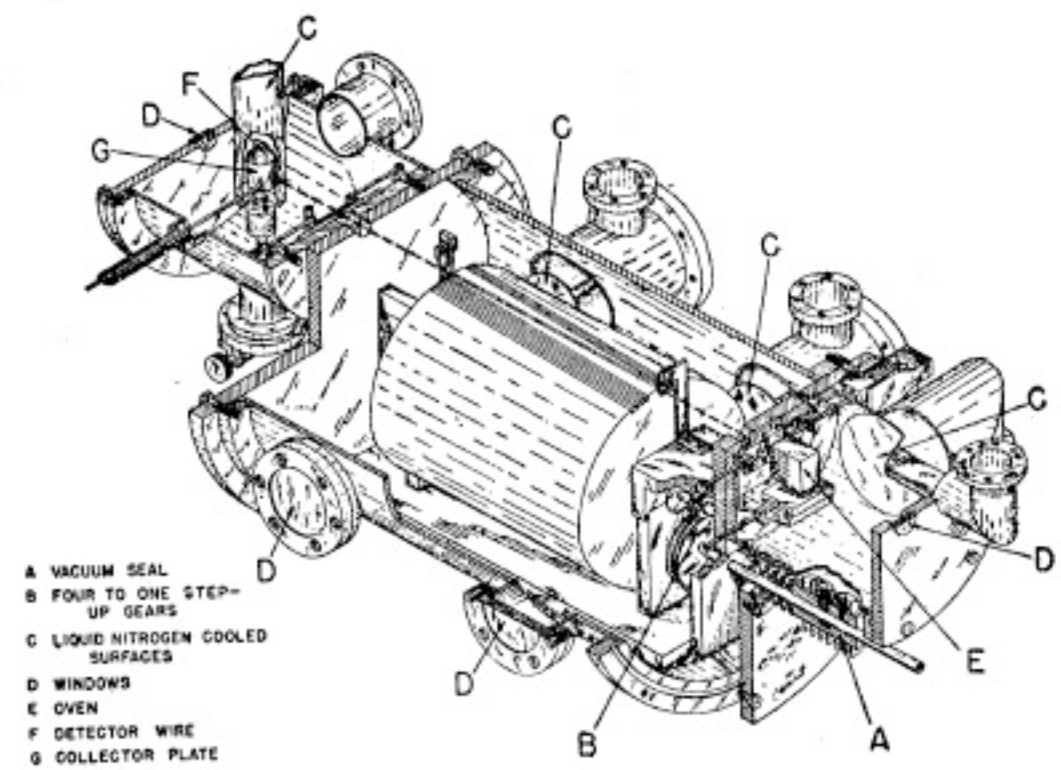


FIG. 2. Schematic diagram of the apparatus designed to measure velocity distributions.

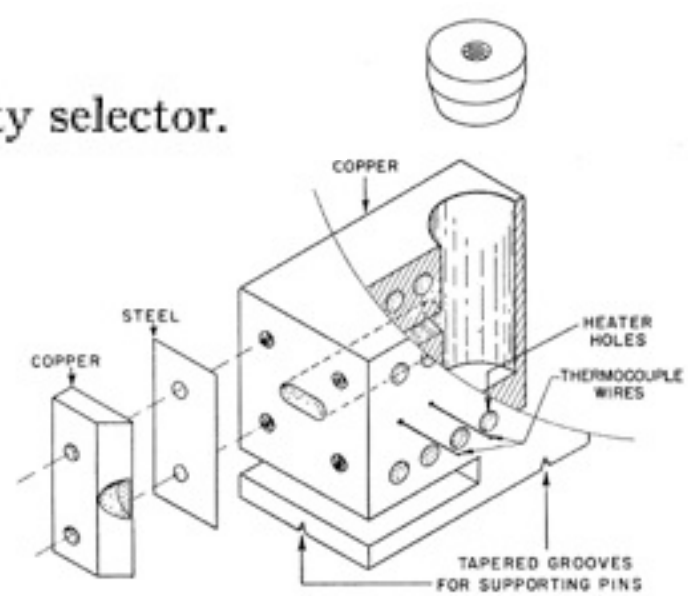


FIG. 3. Copper oven used for potassium showing details of slit construction.

真のオリジナルな実験は手作りの装置からしか生まれない（MY談）

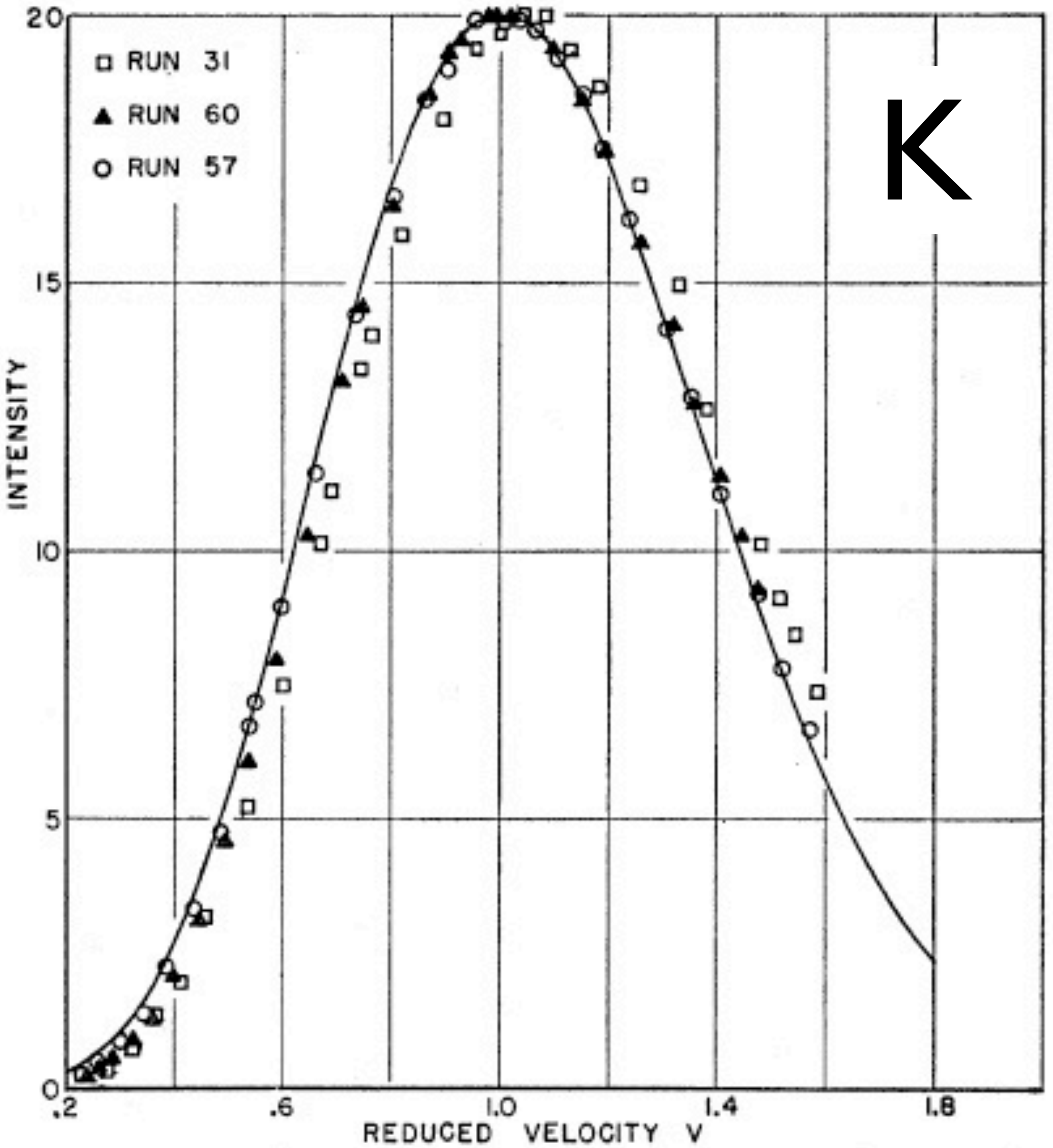


FIG. 4. Typical potassium velocity distributions. The vapor pressures in the ovens are given in Table II. Run 31 was made with thick oven slits and runs 57 and 60 with thin slits.

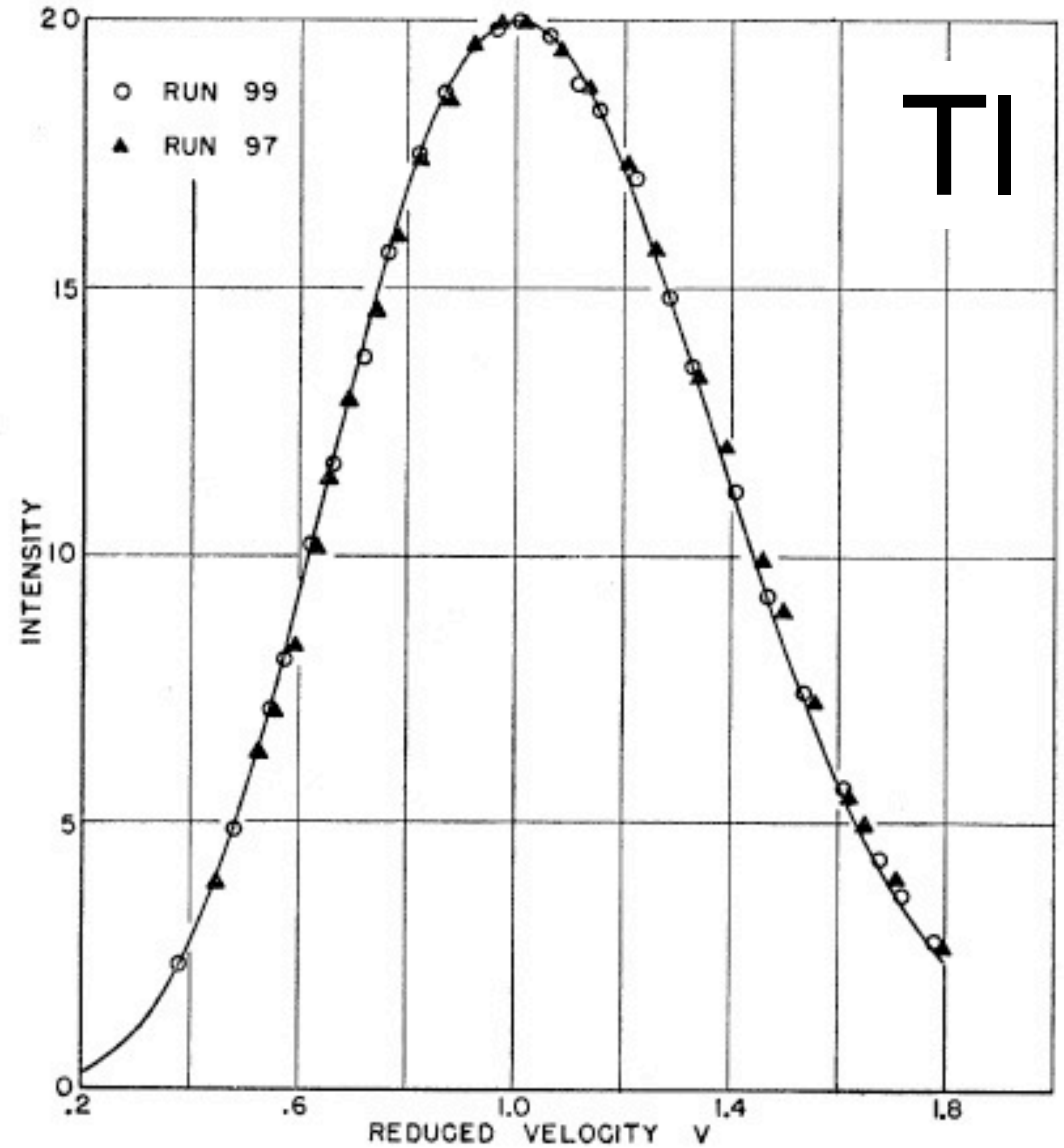


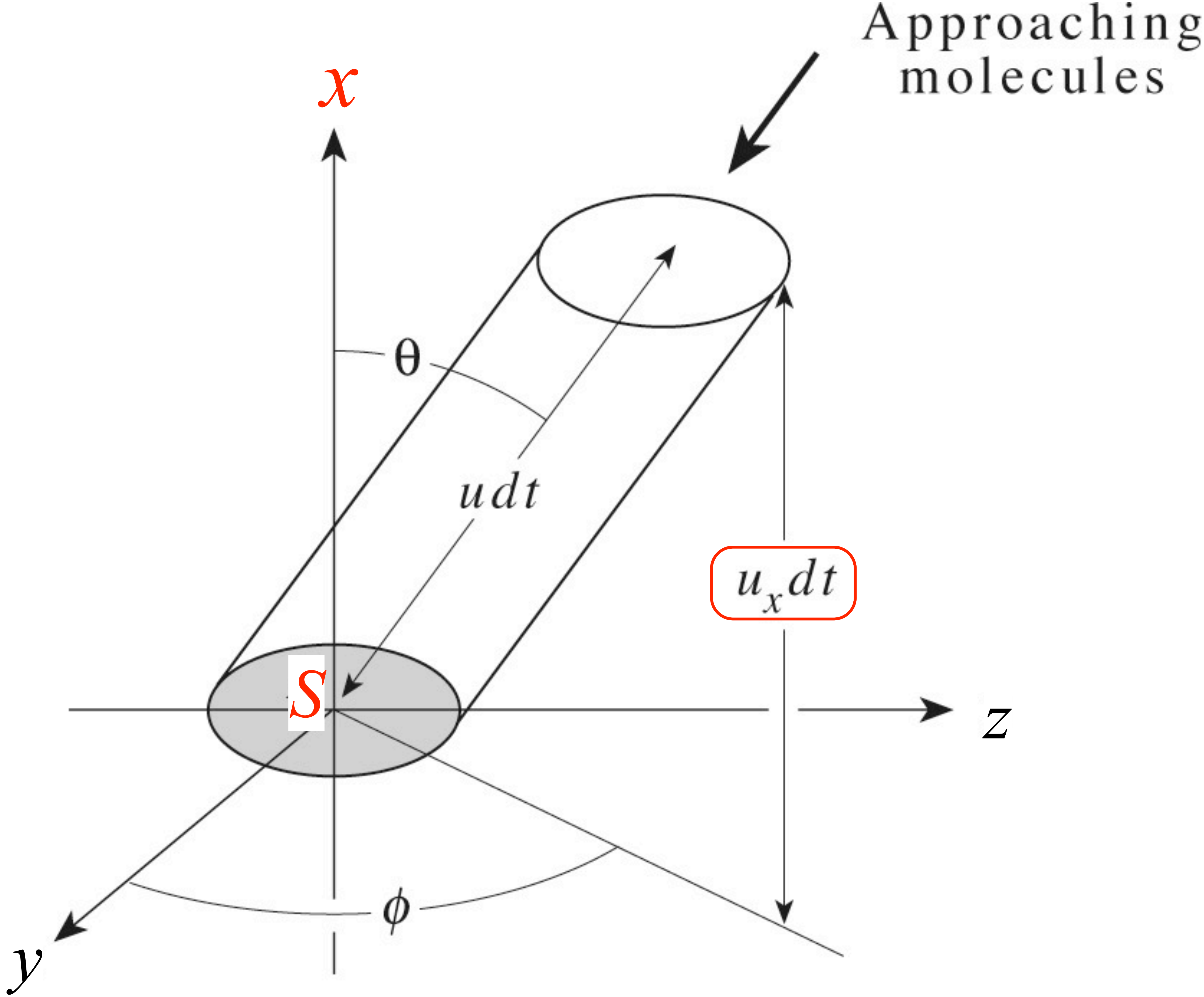
FIG. 5. Typical thallium velocity distributions. The data were taken with thin oven slits at vapor pressures given in Table II.

最大速度をv=1としている

TABLE II. Experimental conditions and results for measured velocity distributions.

Beam	Run	Temperature deg K	Oven pressure in mm of mercury	Velocity of I_{max} in m/sec From oven temp	Velocity of I_{max} in m/sec From exp distr
K	57	466±2	4.5×10^{-3}	628±2	630±3
K	60	544±3	1.2×10^{-1}	678±3	679±3
K	31	489±2	1.4×10^{-2}	644±2	682±3
Tl	99	870±4	3.2×10^{-3}	376±1	376±2
Tl	97	944±5	2.1×10^{-2}	392±1	395±2

Fig.27.5



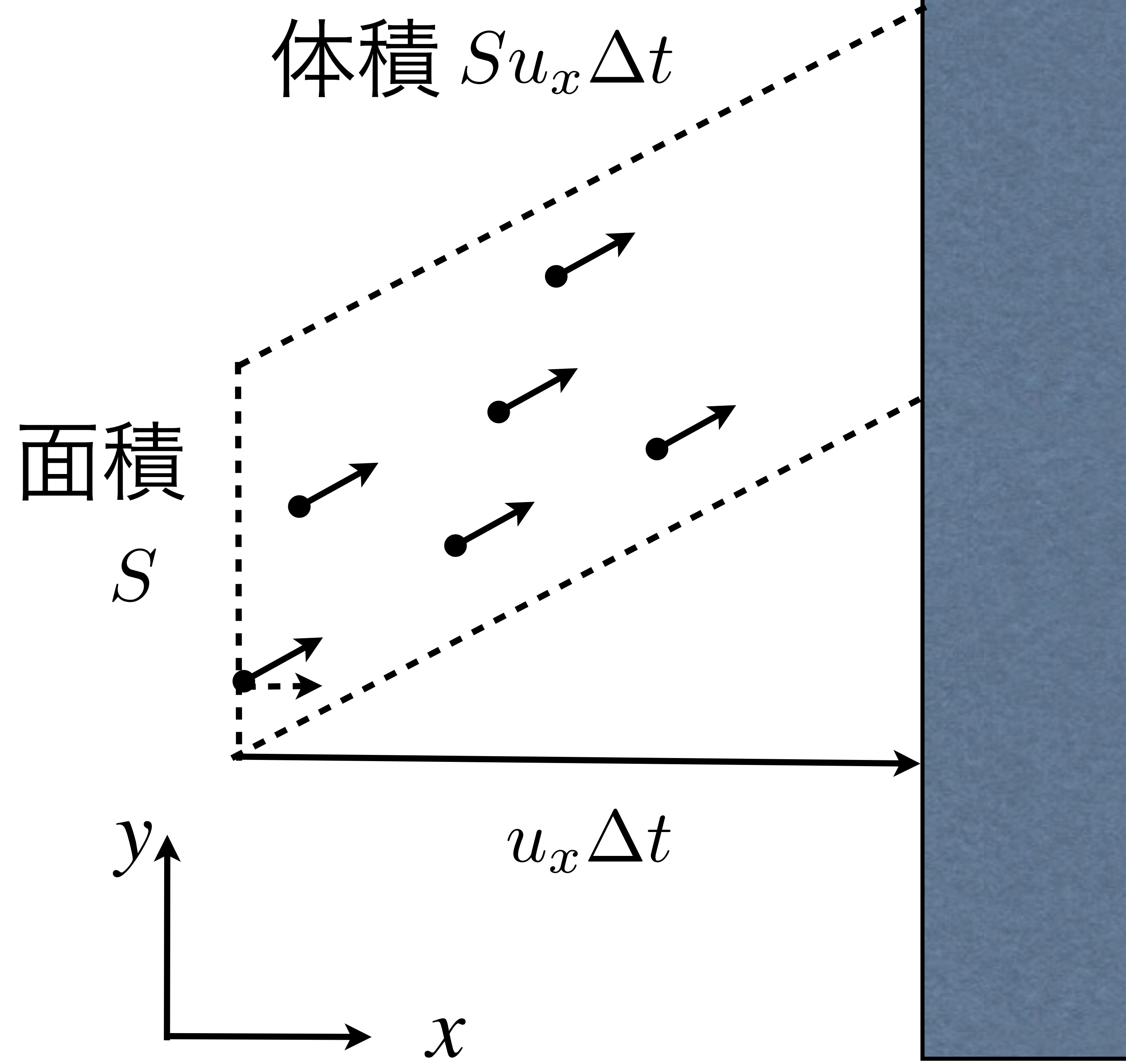
$$h(u_x, u_y, u_z)du_x du_y du_z$$

$$u_x \sim u_x + du_x$$

$$u_y \sim u_y + du_y$$

$$u_z \sim u_z + du_z$$

の間に速度成分を持つ確率



$$(Su_x\Delta t)\frac{N}{V}h(u_x, u_y, u_z)du_x du_y du_z$$

密度ρ=N /V

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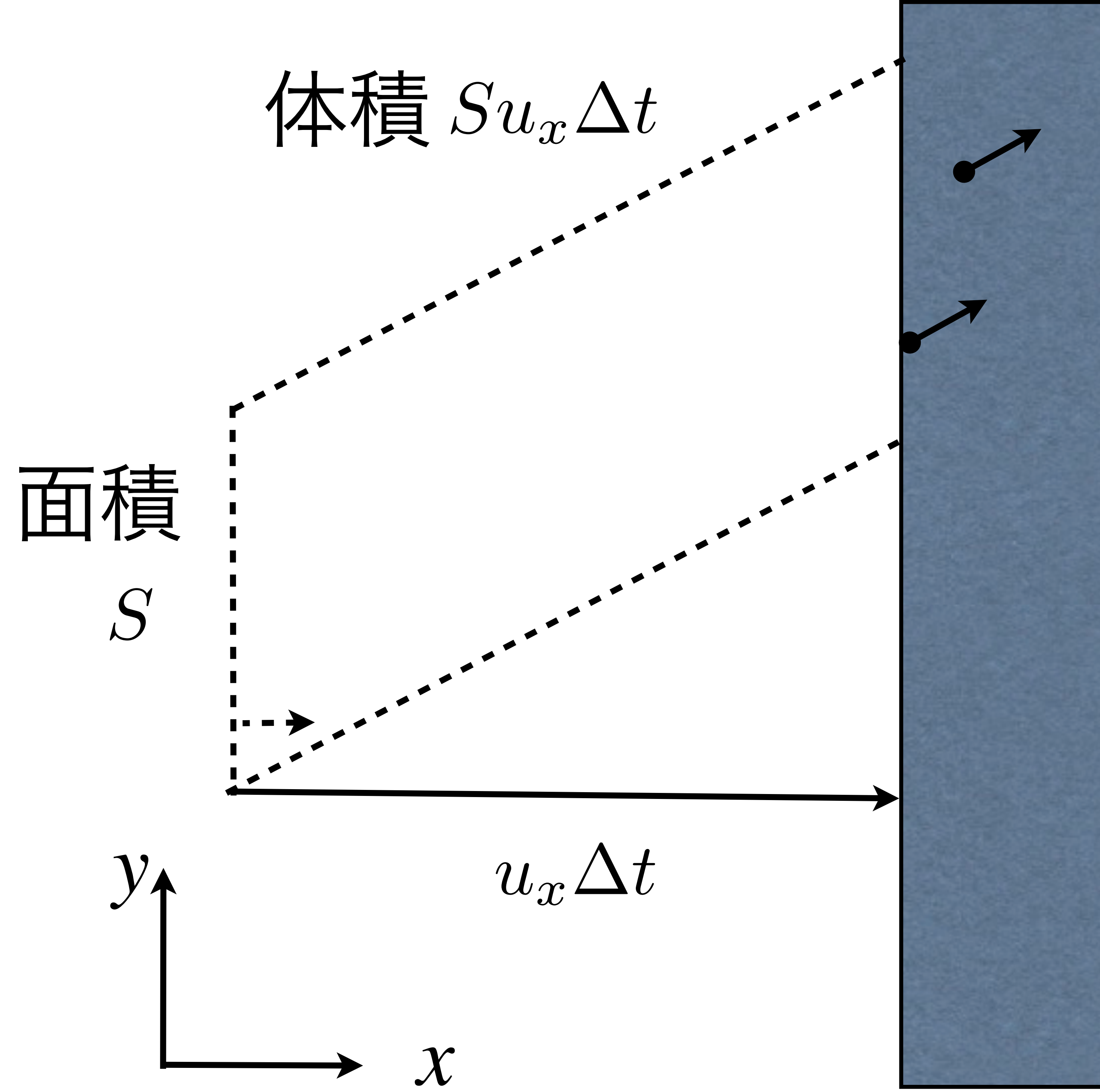

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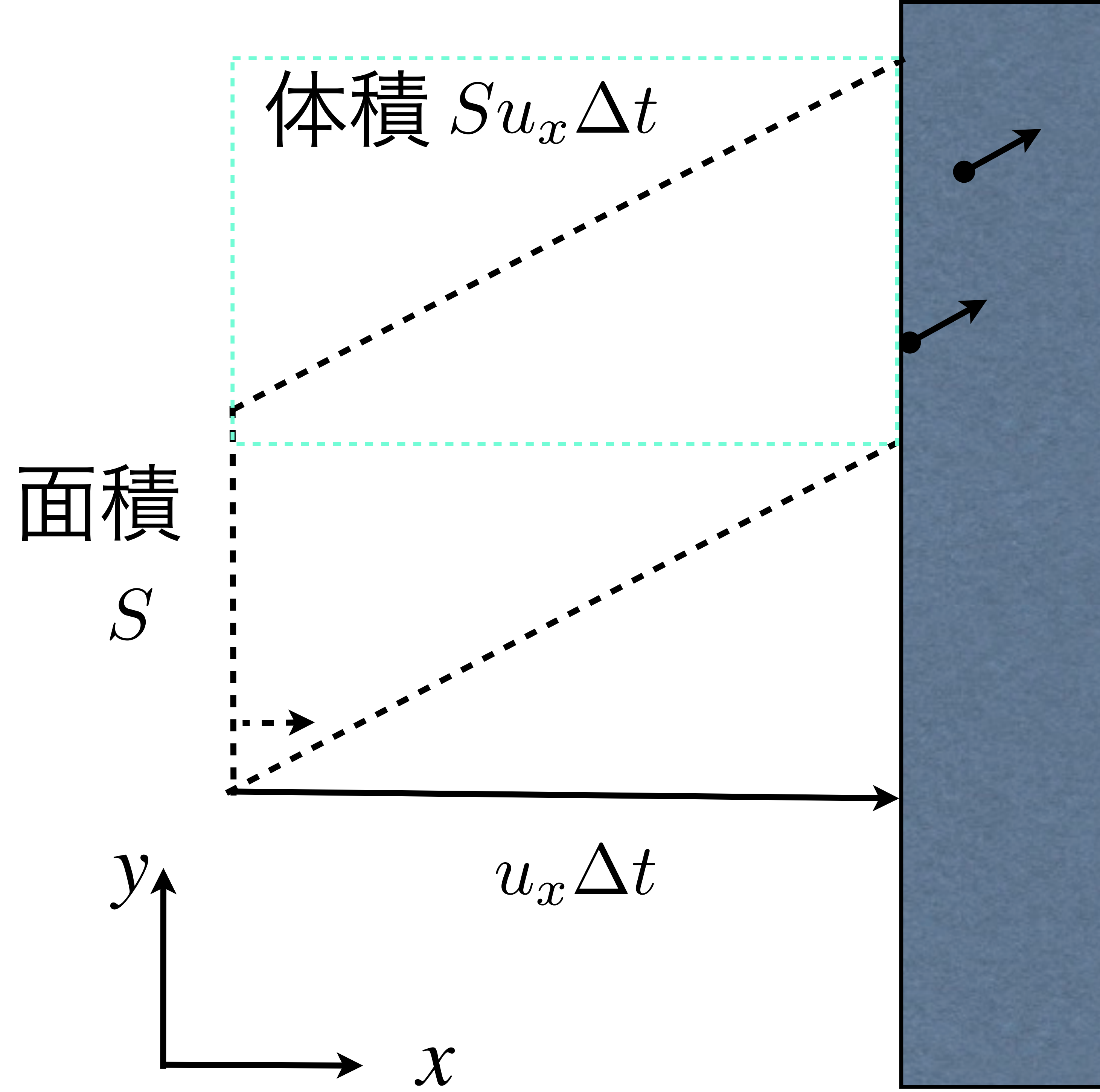

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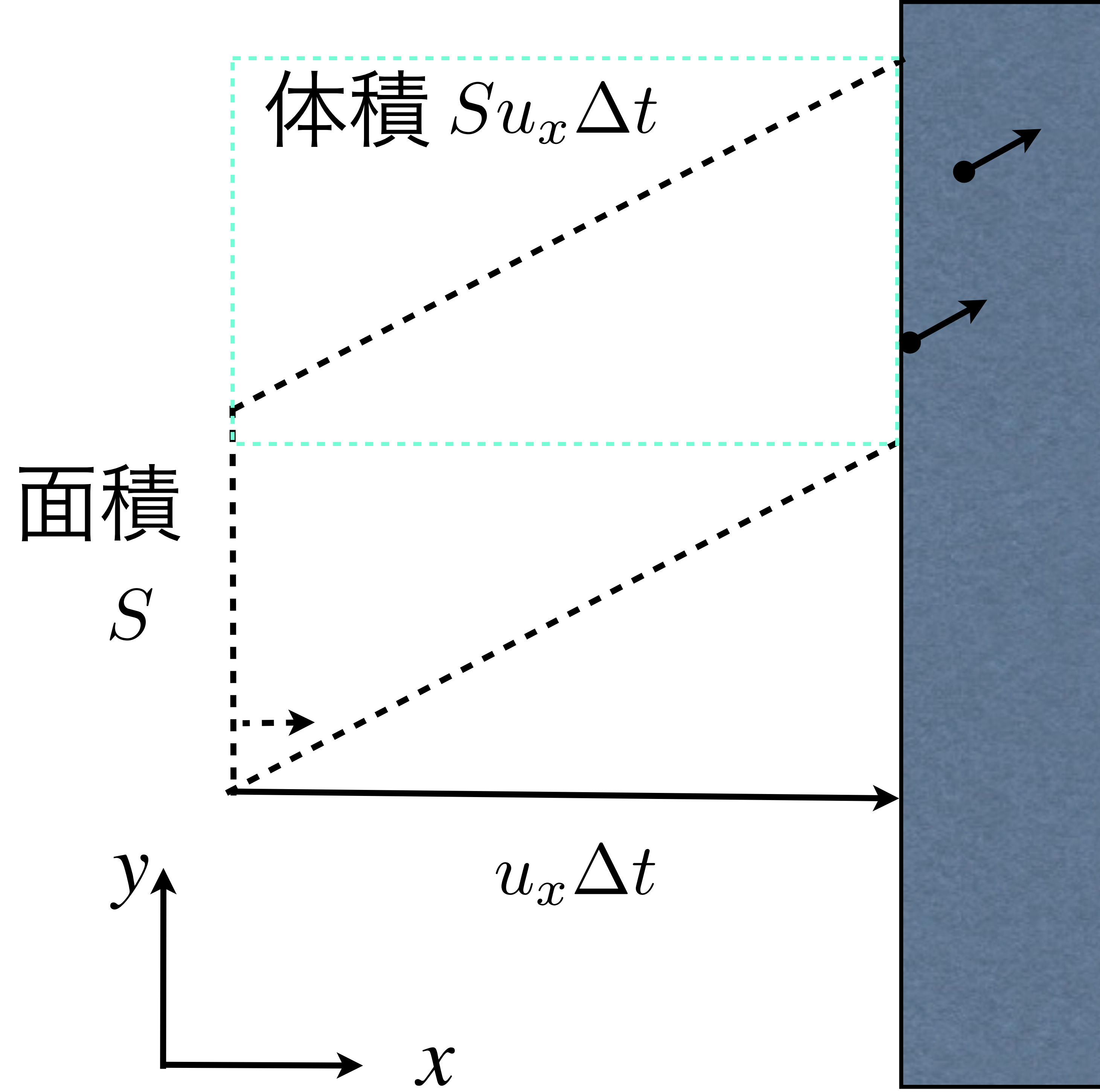

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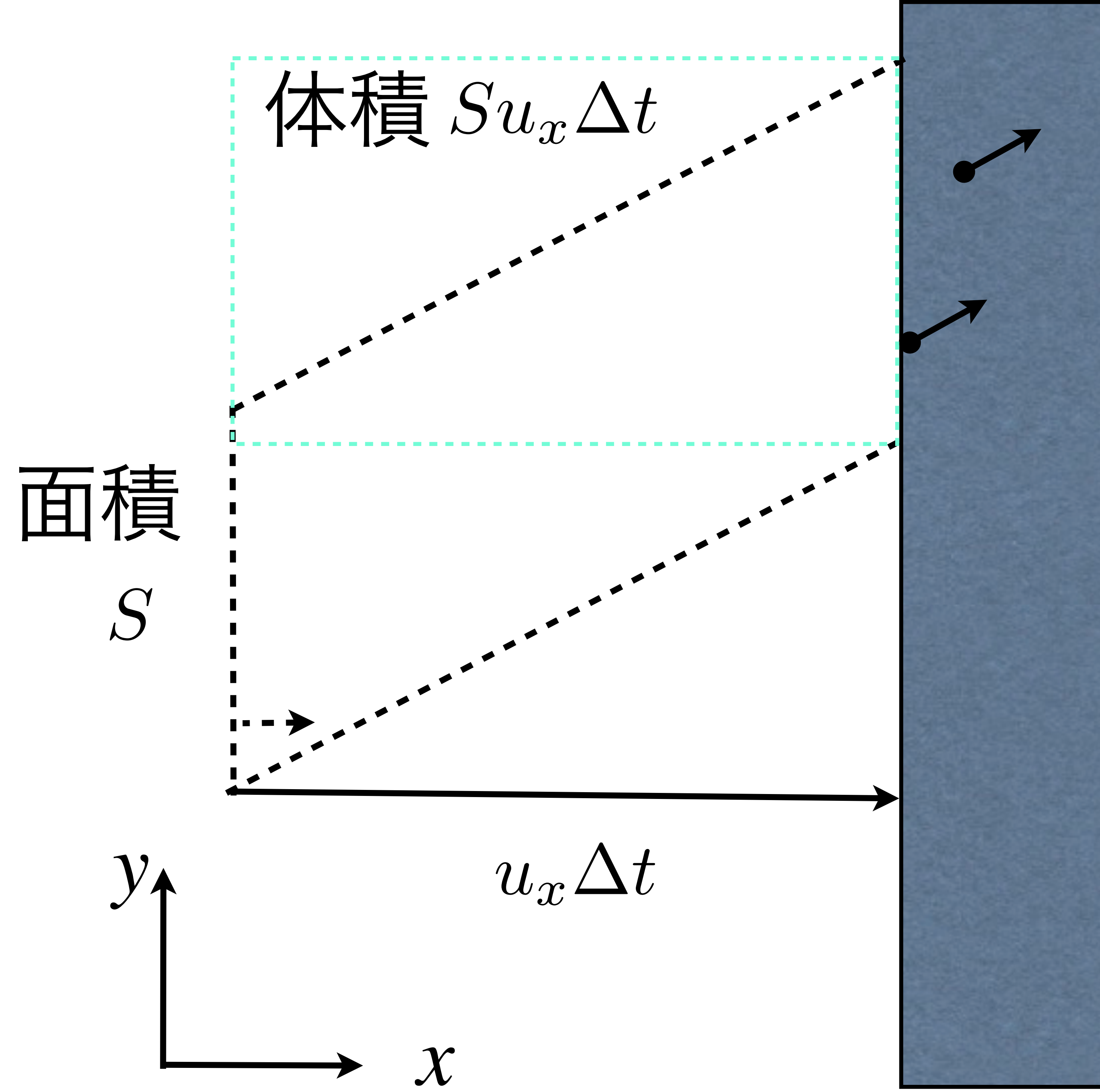

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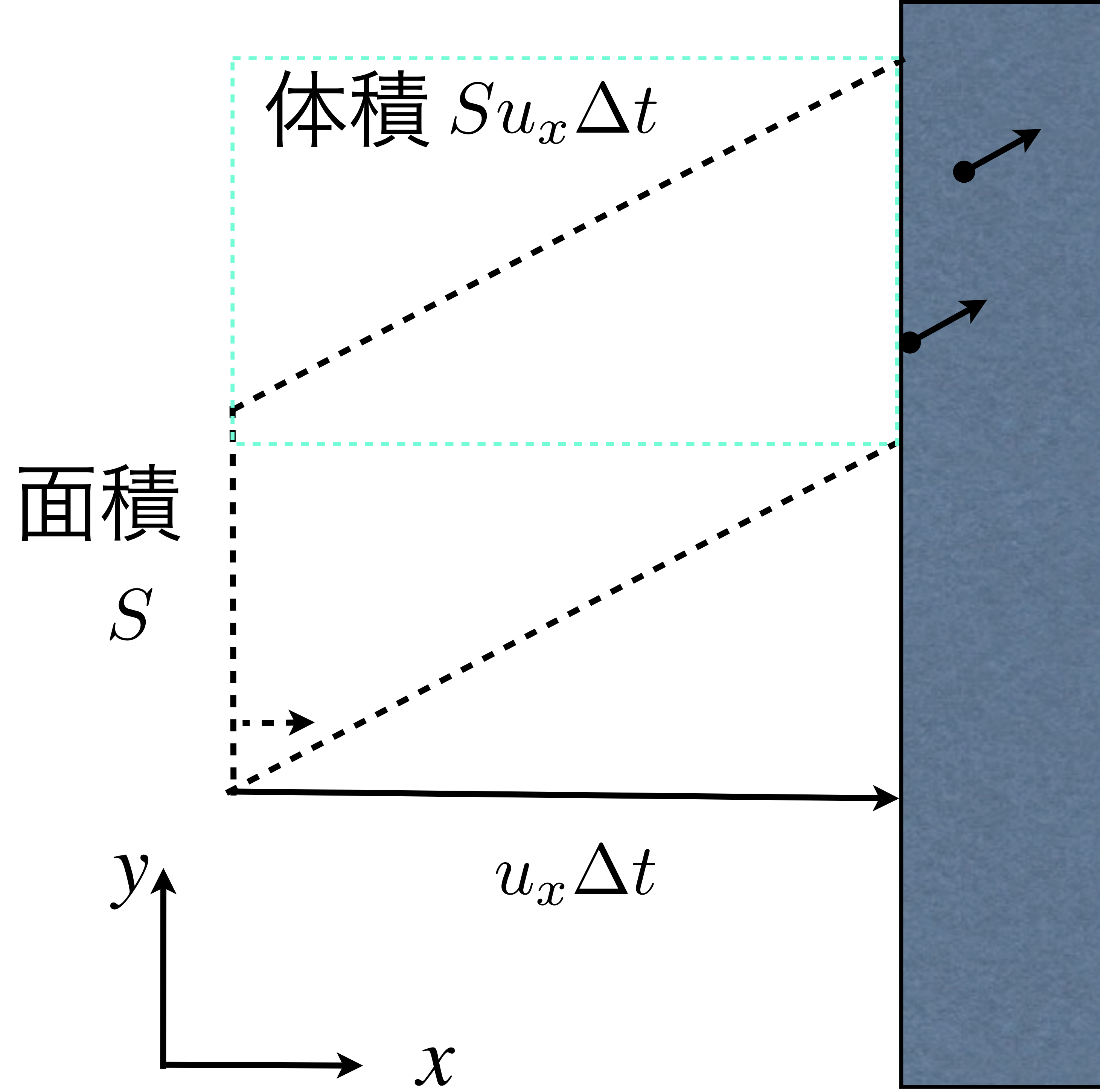

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密度ρ=N / V

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

Newtonの運動方程式

$$\int_{t_1}^{t_2} m\frac{d\vec{v}}{dt}dt = \int_{t_1}^{t_2} \vec{F}dt$$

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$$\int_{t_1}^{t_2} \vec{F}dt \simeq \langle\vec{F}\rangle\Delta t, \quad \Delta t = t_2 - t_1$$

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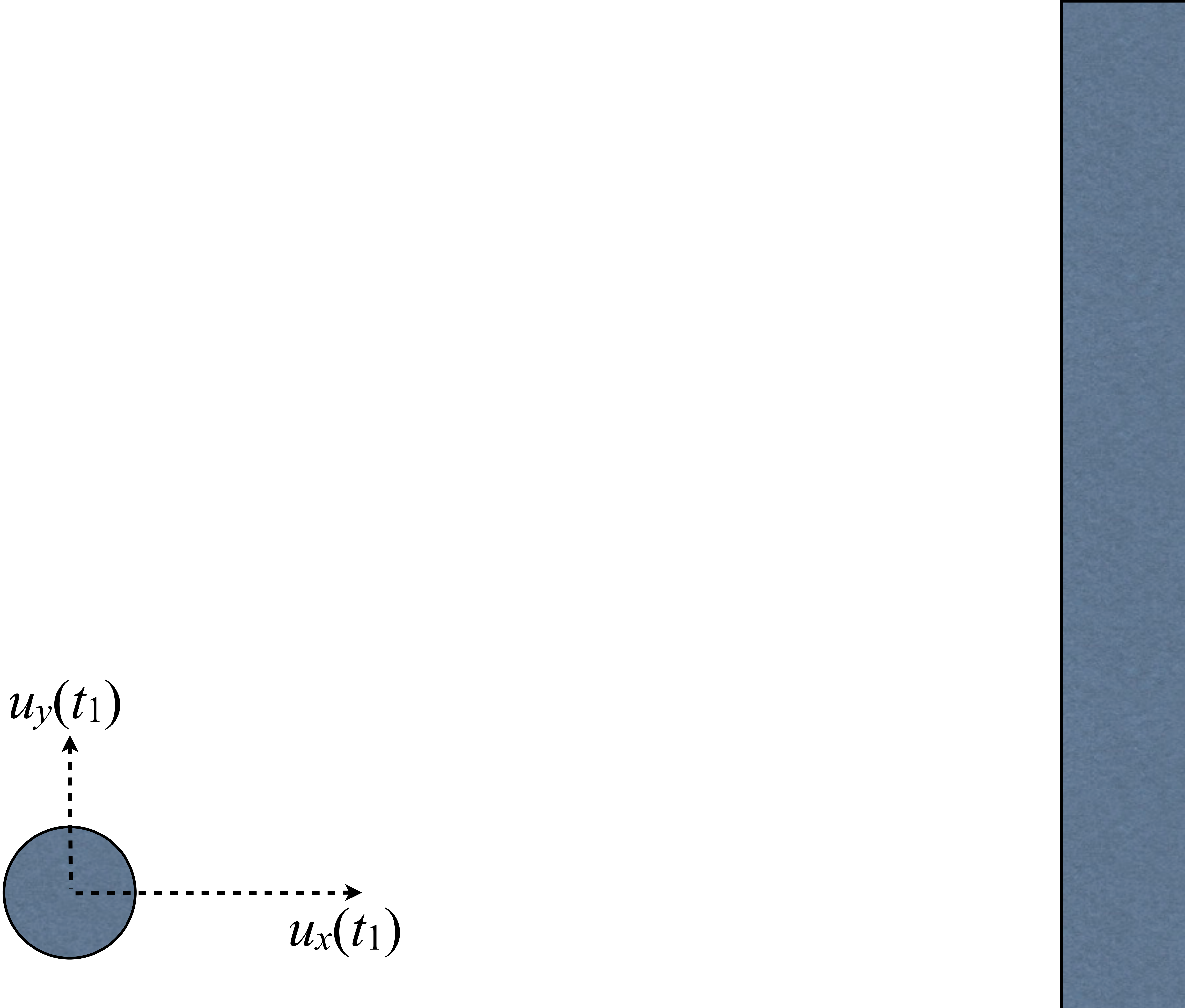
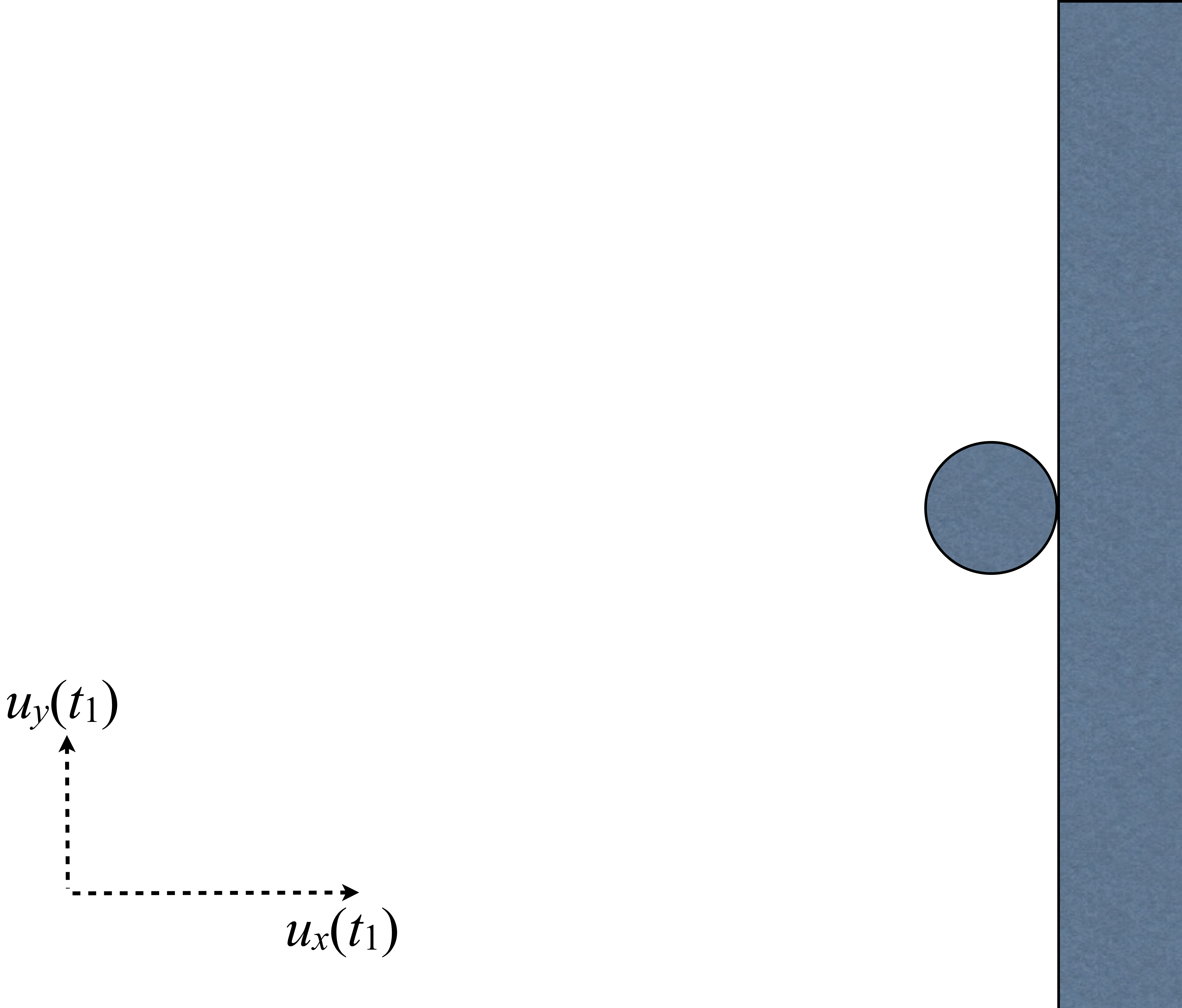

$u_y(t_1)$
$u_x(t_1)$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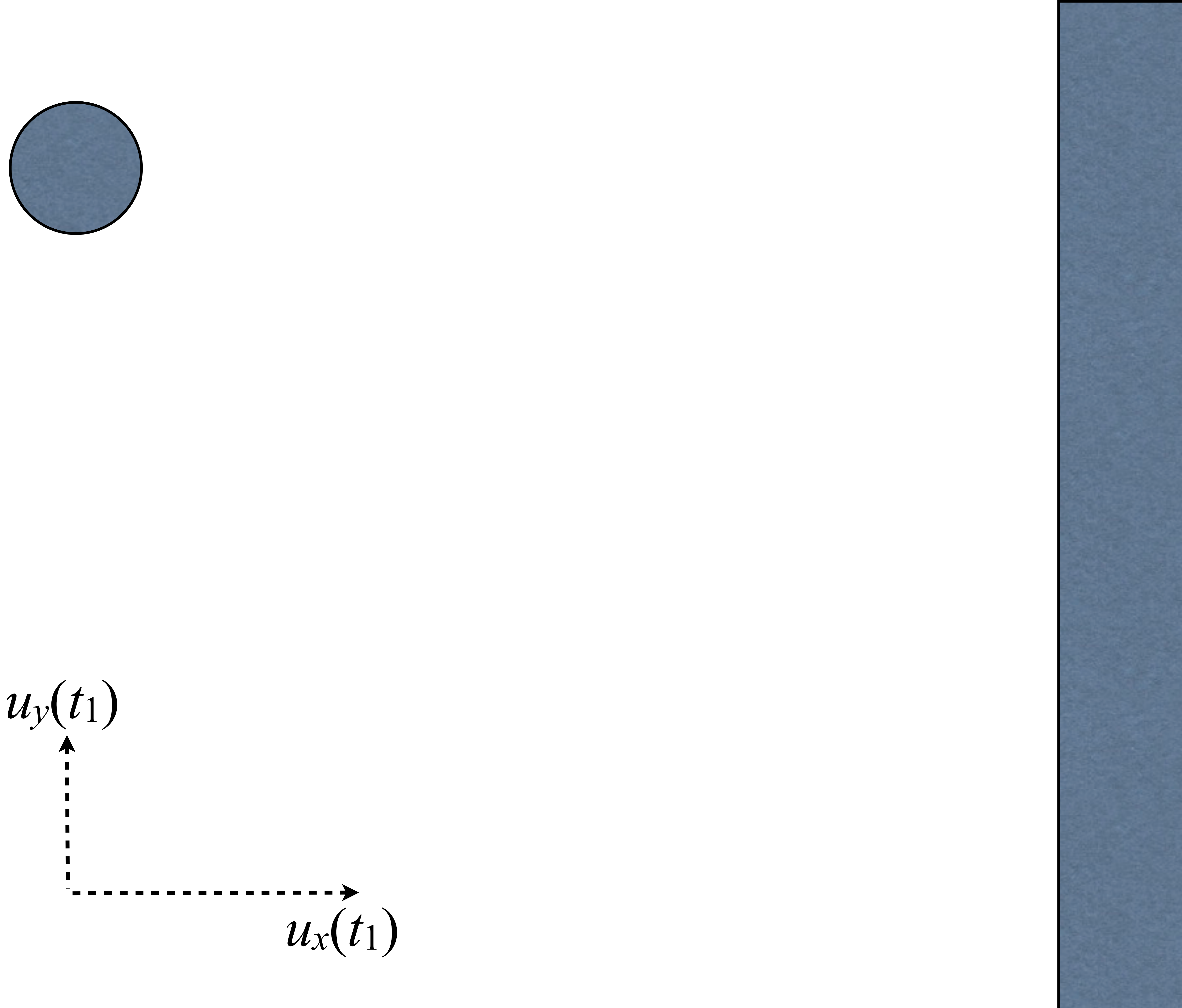

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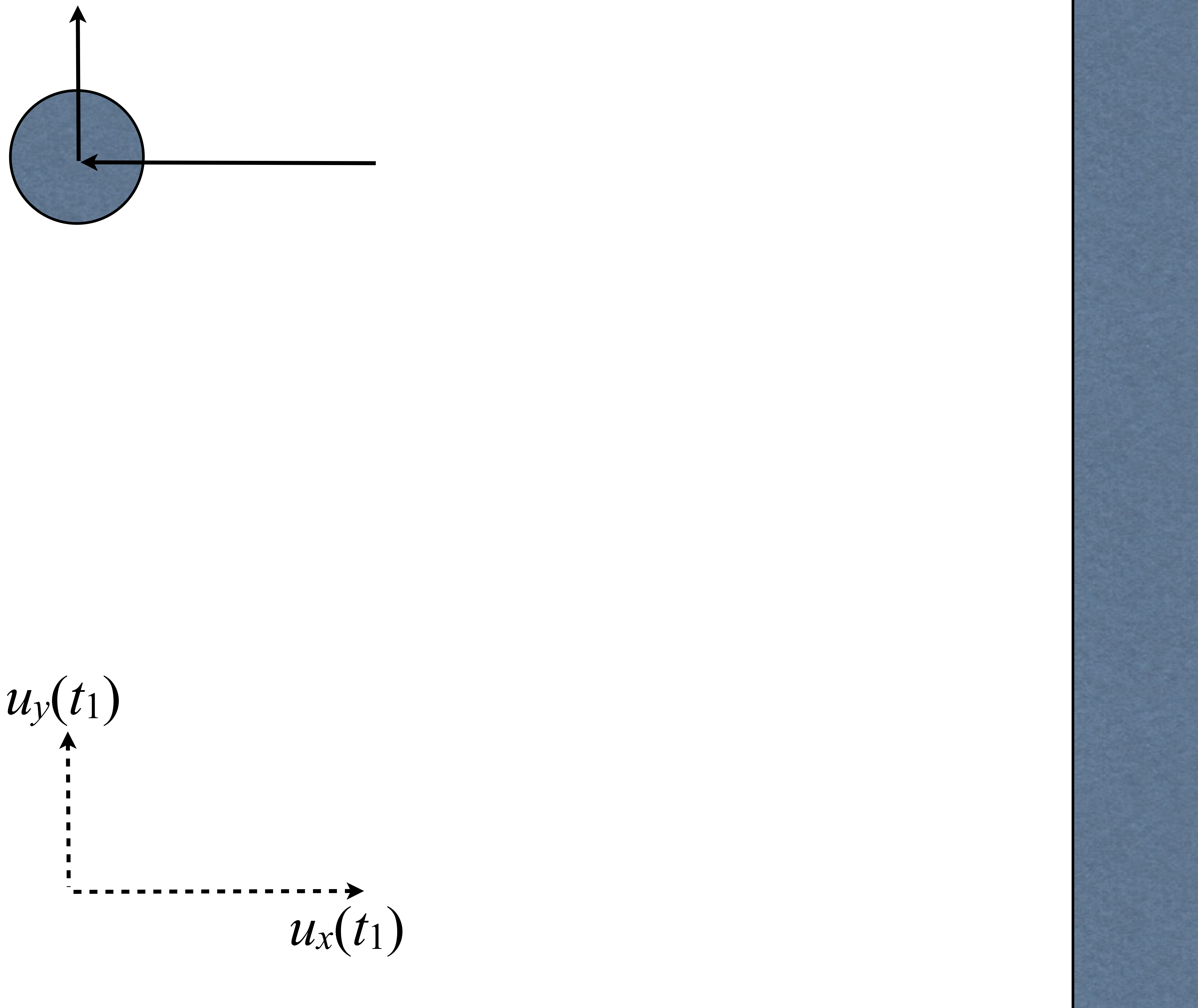
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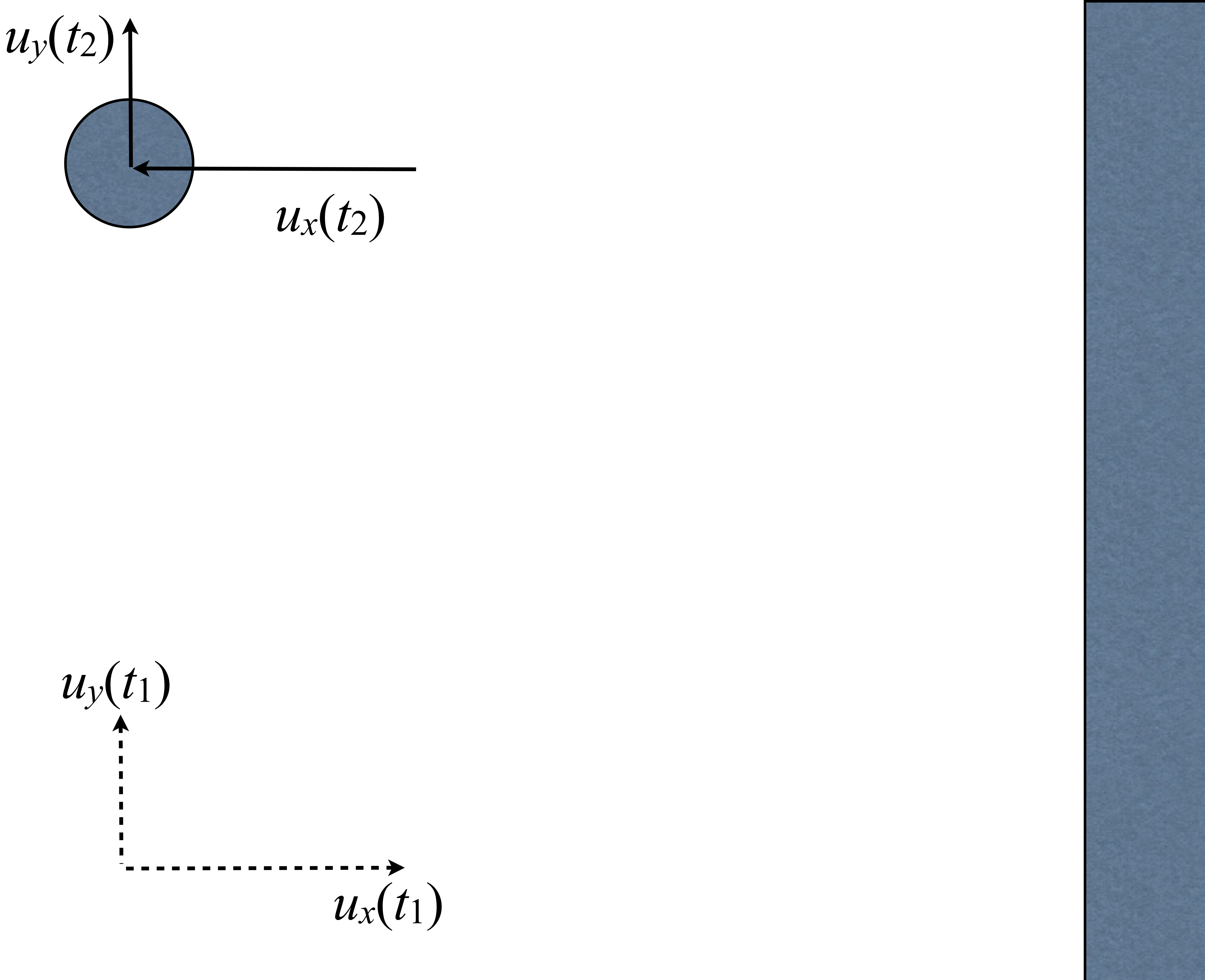
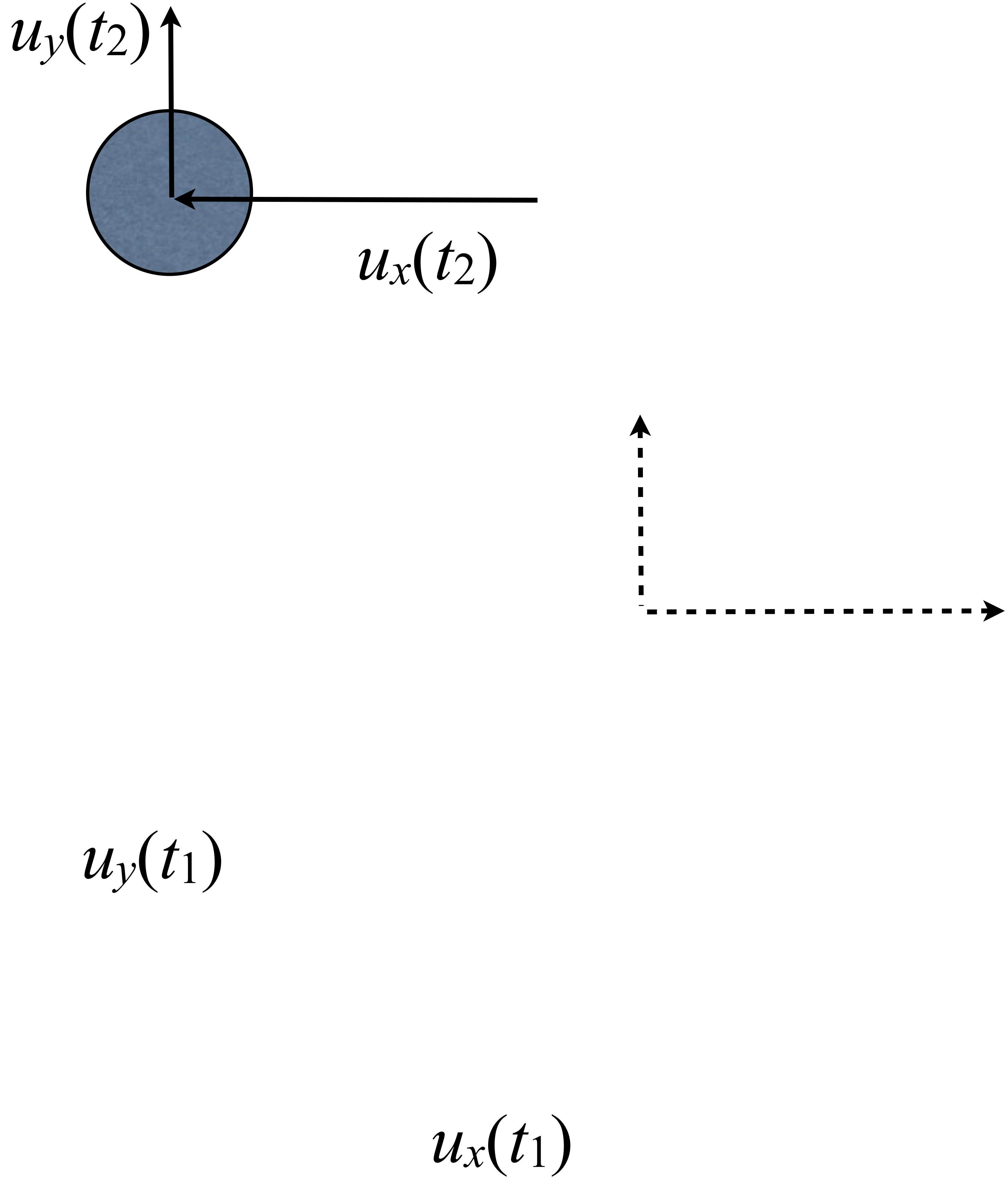


$u_y(t_2)$
$u_x(t_2)$
$u_y(t_1)$
$u_x(t_1)$



$u_y(t_2)$
$u_x(t_2)$
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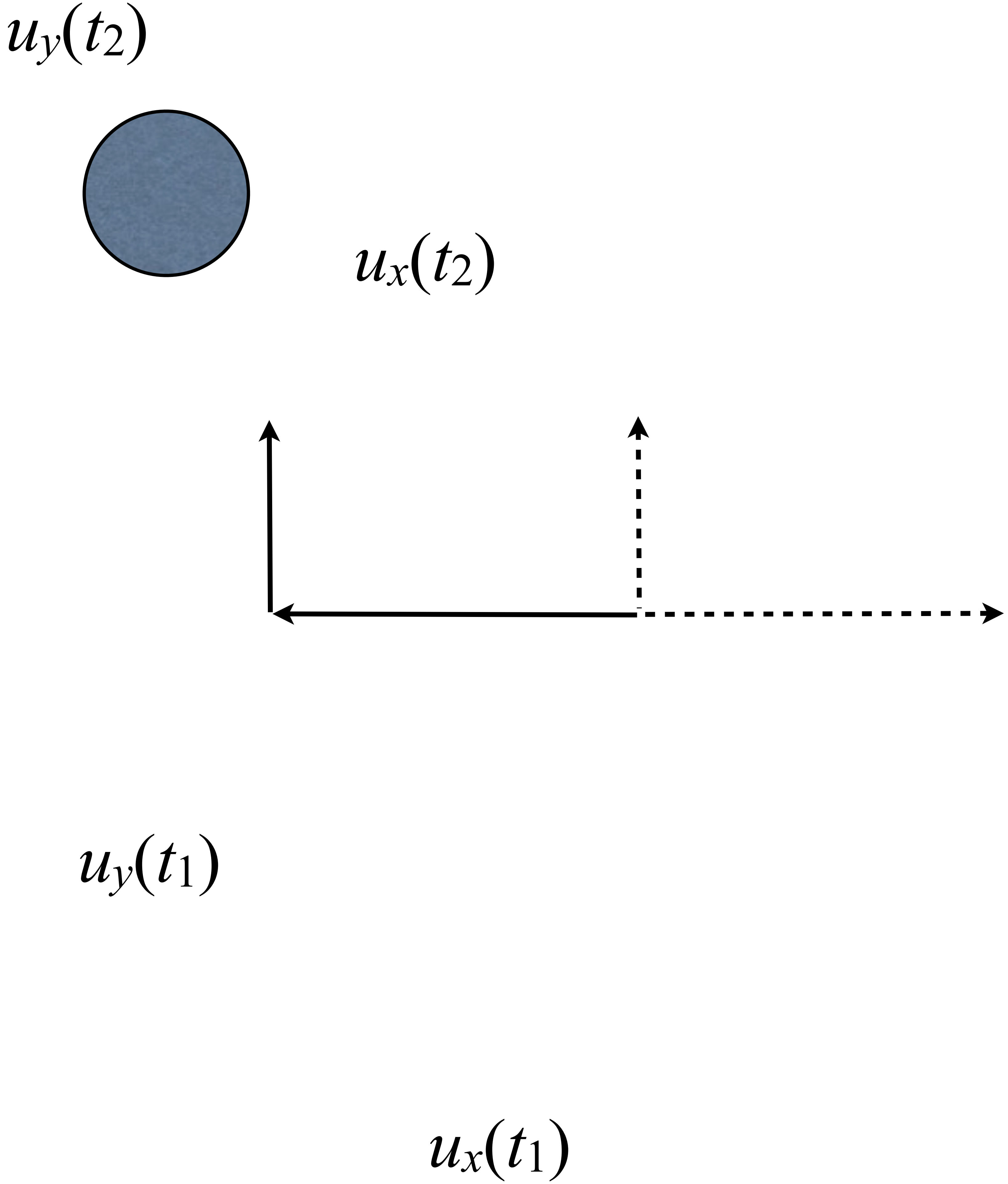
$u_y(t_2)$
$u_x(t_2)$
$u_y(t_1)$
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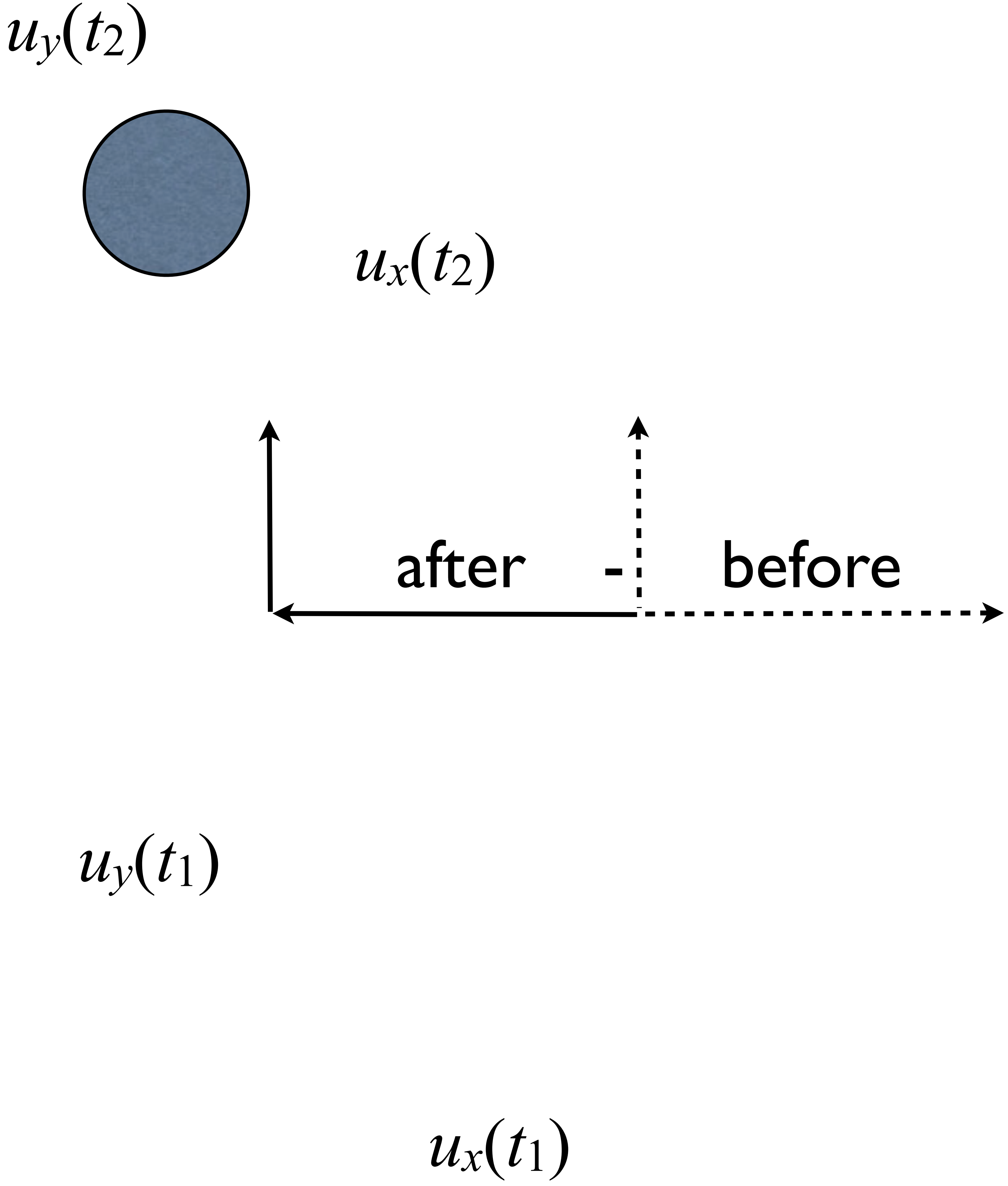
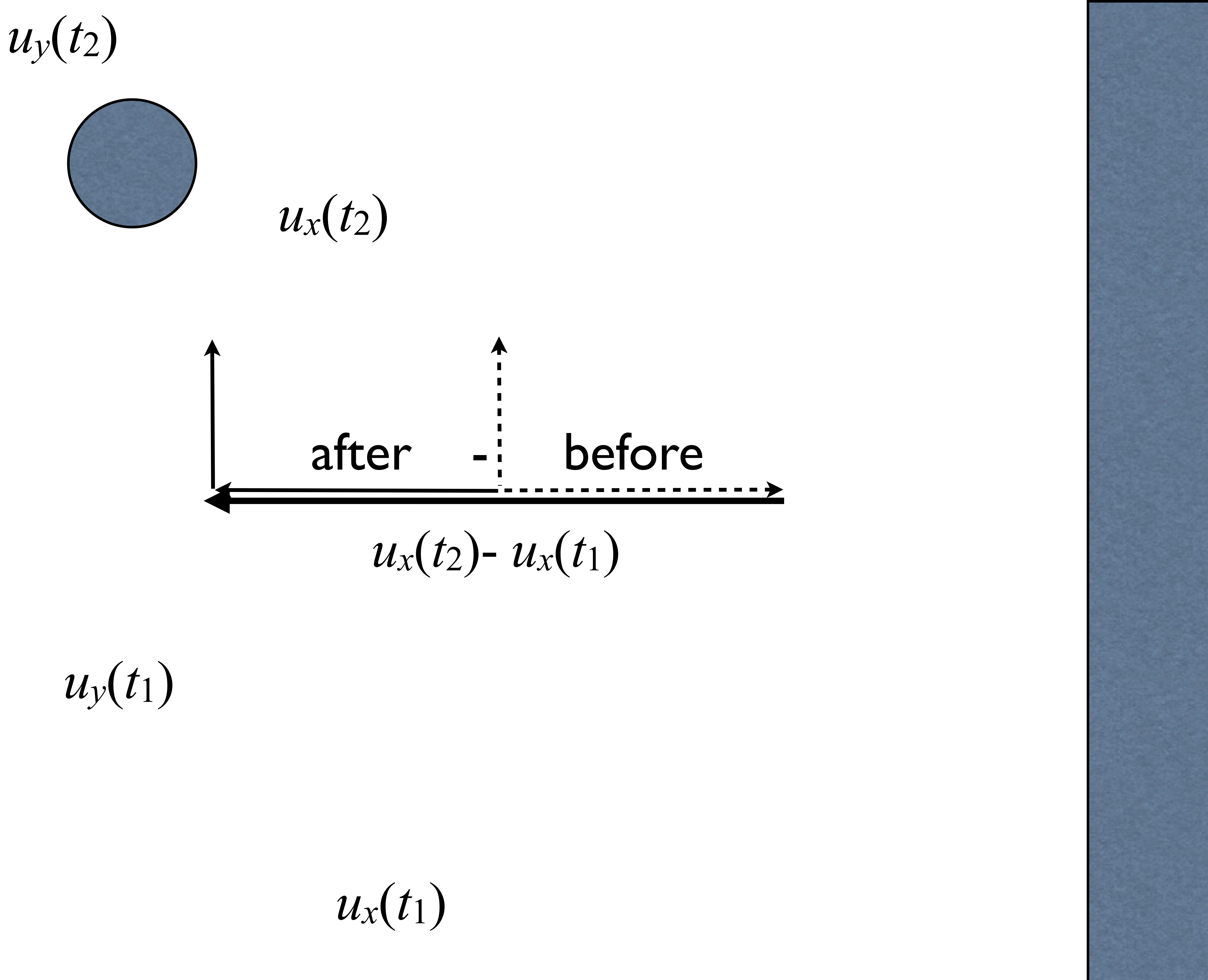
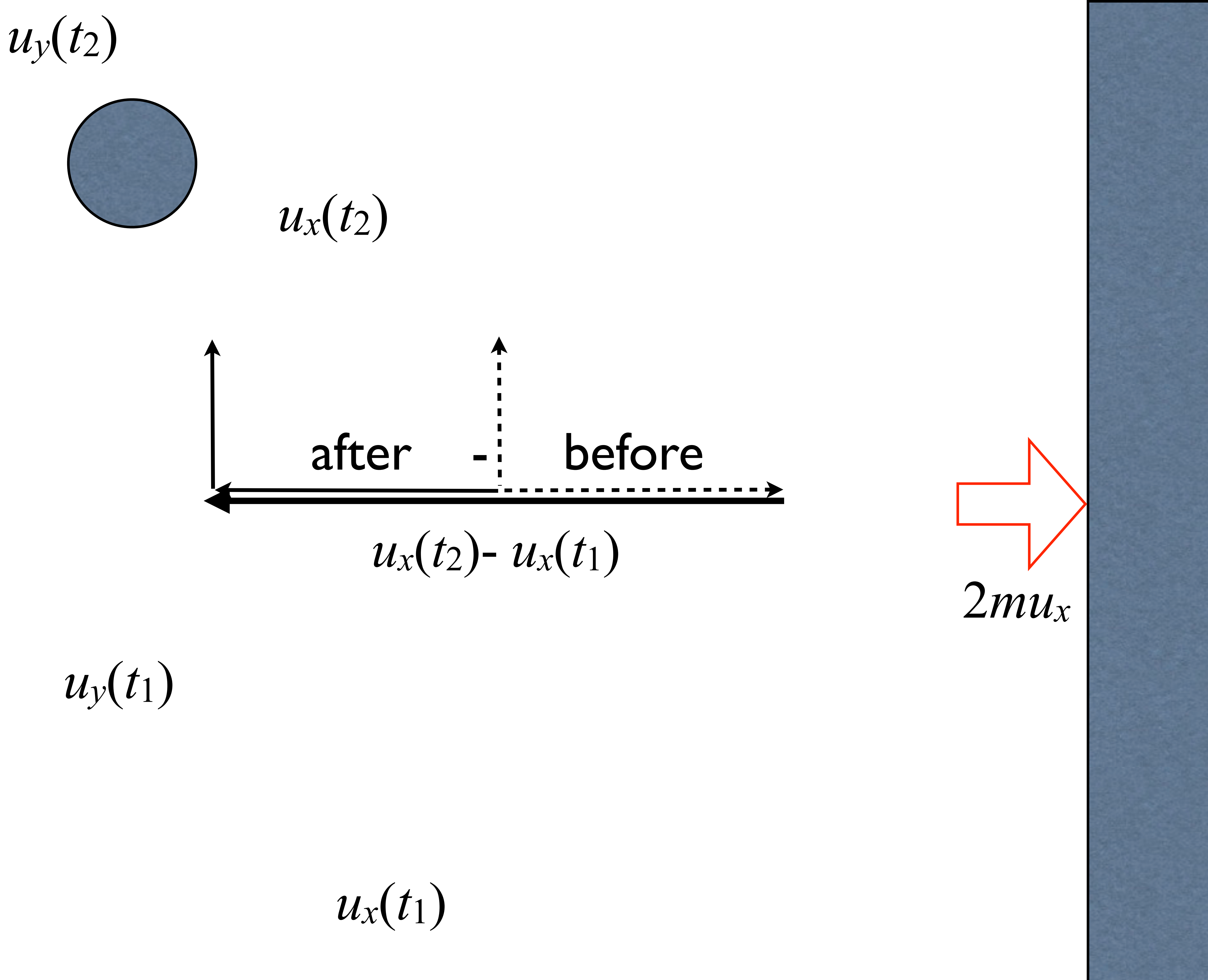

$u_y(t_2)$
$u_x(t_2)$
after
-
before
$u_y(t_1)$
$u_x(t_1)$


$u_y(t_2)$
$u_x(t_2)$
after
-
before
$u_x(t_2)$- $u_x(t_1)$
$u_y(t_1)$
$u_x(t_1)$



$u_y(t_2)$
$u_x(t_2)$
after
-
before
$u_x(t_2)$- $u_x(t_1)$
$2mu_x$
$u_y(t_1)$
$u_x(t_1)$

$$P = \frac{1}{S\Delta t}\int_0^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z (Su_x\Delta t)\frac{N}{V}h(u_x, u_y, u_z)2mu_x$$

$$\begin{aligned} PV &= N\int_0^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2mu_x^2 h(u_x, u_y, u_z) \\ &= \frac{N}{2}\int_{-\infty}^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2mu_x^2 h(u_x, u_y, u_z) \\ &= Nm\langle u_x^2\rangle \end{aligned}$$

$$\begin{aligned} u^2 &= u_x^2 + u_y^2 + u_z^2 \\ \langle u_x^2\rangle &= \langle u_y^2\rangle = \langle u_z^2\rangle \\ \langle u^2\rangle &= 3\langle u_x^2\rangle \end{aligned}$$

$$PV = \frac{1}{3}Nm\langle u^2\rangle = \frac{2}{3}N\frac{m}{2}\langle u^2\rangle = \frac{2}{3}E_{\text{kinetic}}$$

$$P = \frac{1}{S\Delta t}\int_0^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z (S u_x \Delta t)\frac{N}{V} h(u_x, u_y, u_z) 2 m u_x$$

$$\begin{aligned} PV &= N\int_0^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2 m u_x^2 h(u_x, u_y, u_z) \\ &= \frac{N}{2}\int_{-\infty}^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2 m u_x^2 h(u_x, u_y, u_z) \\ &= N m \langle u_x^2 \rangle \end{aligned}$$

$$u^2 = u_x^2 + u_y^2 + u_z^2$$

$$\langle u_x^2 \rangle = \langle u_y^2 \rangle = \langle u_z^2 \rangle$$

$$\langle u^2 \rangle = 3\langle u_x^2 \rangle$$

$$PV = \frac{1}{3} N m \langle u^2 \rangle = \frac{2}{3} N \frac{m}{2} \langle u^2 \rangle = \frac{2}{3} E_{\text{kinetic}}$$

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$$\begin{aligned} PV &= N\int_0^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2mu_x^2 h(u_x, u_y, u_z) \\ &= \frac{N}{2}\int_{-\infty}^\infty du_x \int_{-\infty}^\infty du_y \int_{-\infty}^\infty du_z 2mu_x^2 h(u_x, u_y, u_z) \\ &= Nm\langle u_x^2\rangle \end{aligned}$$

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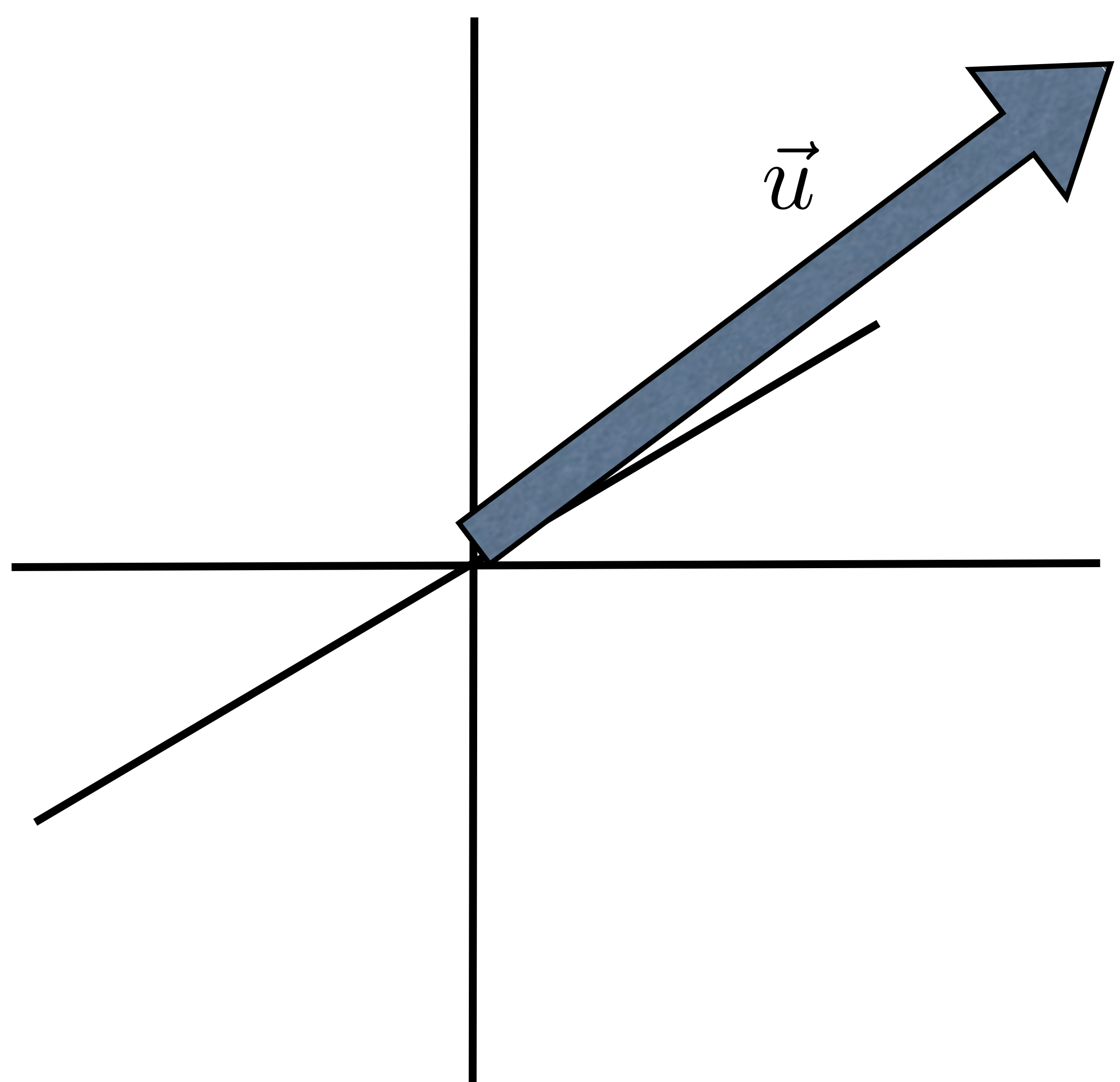
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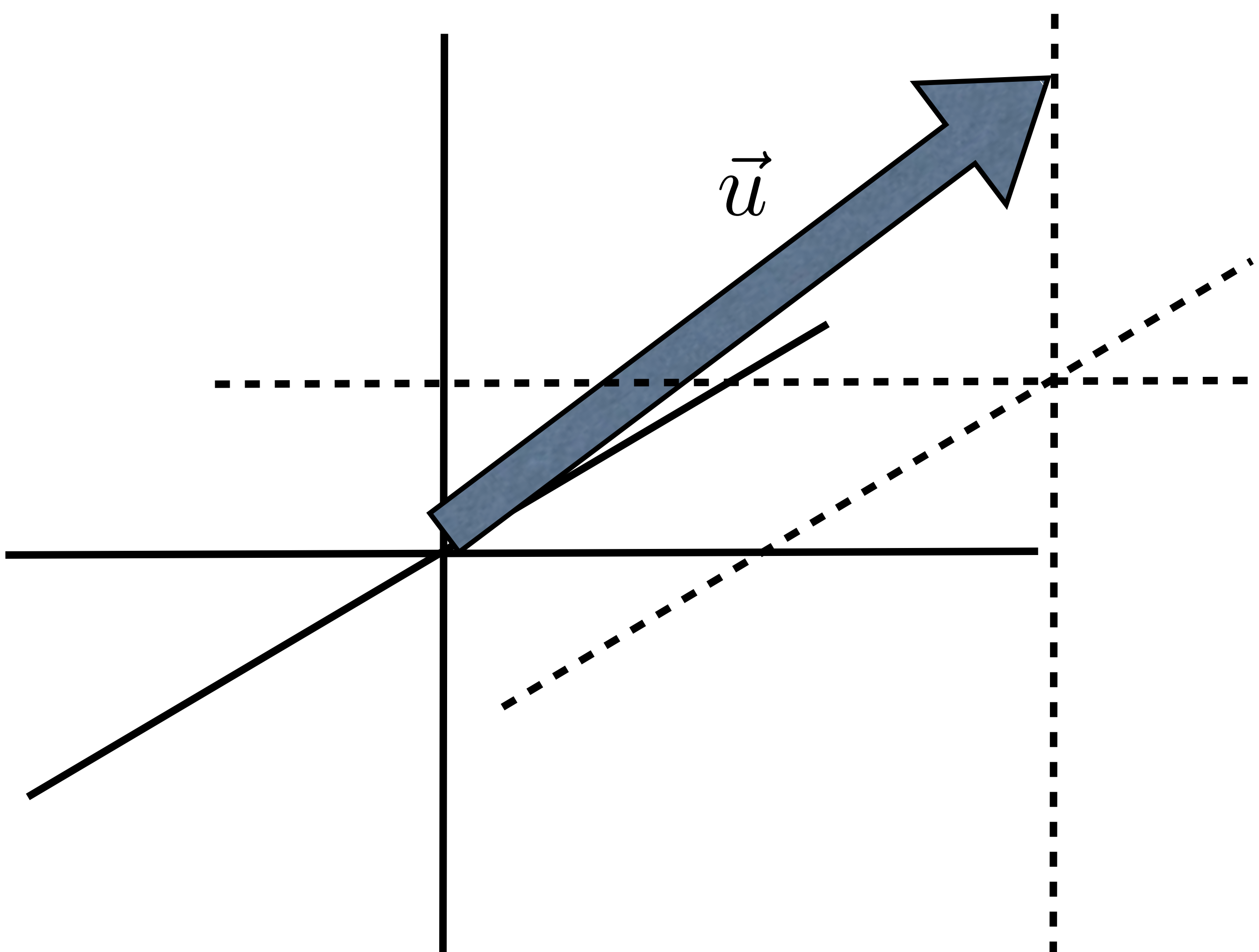
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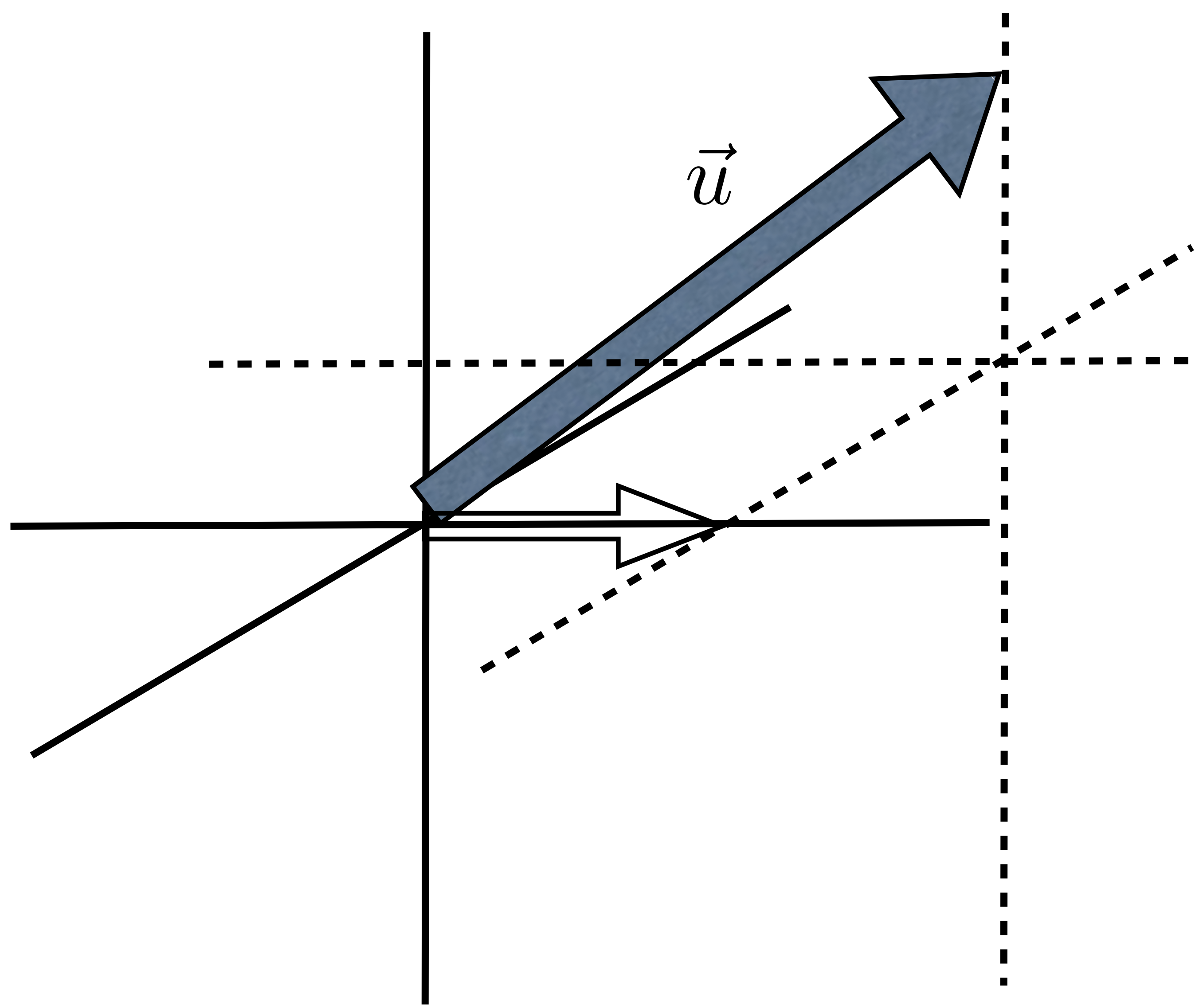
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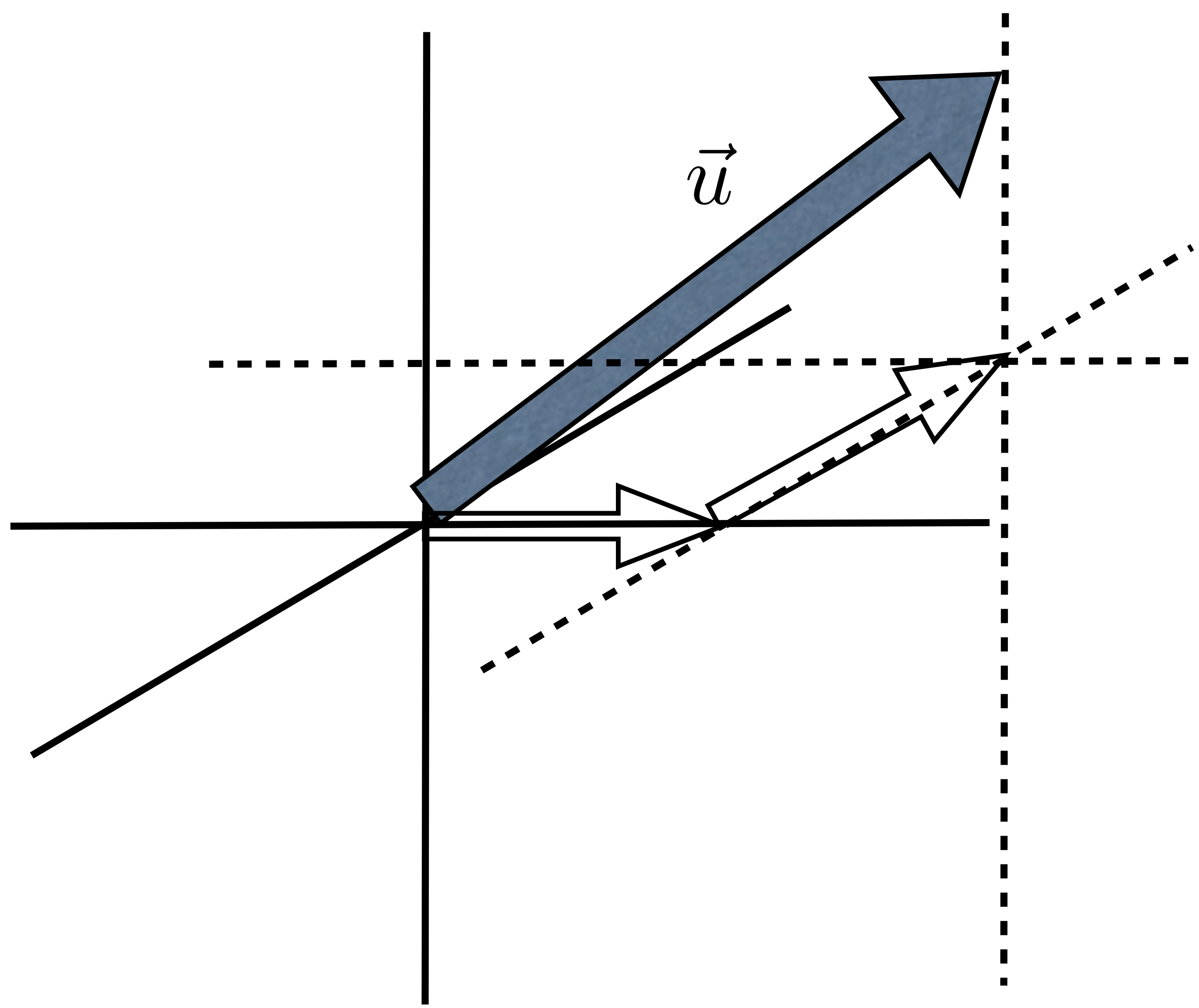
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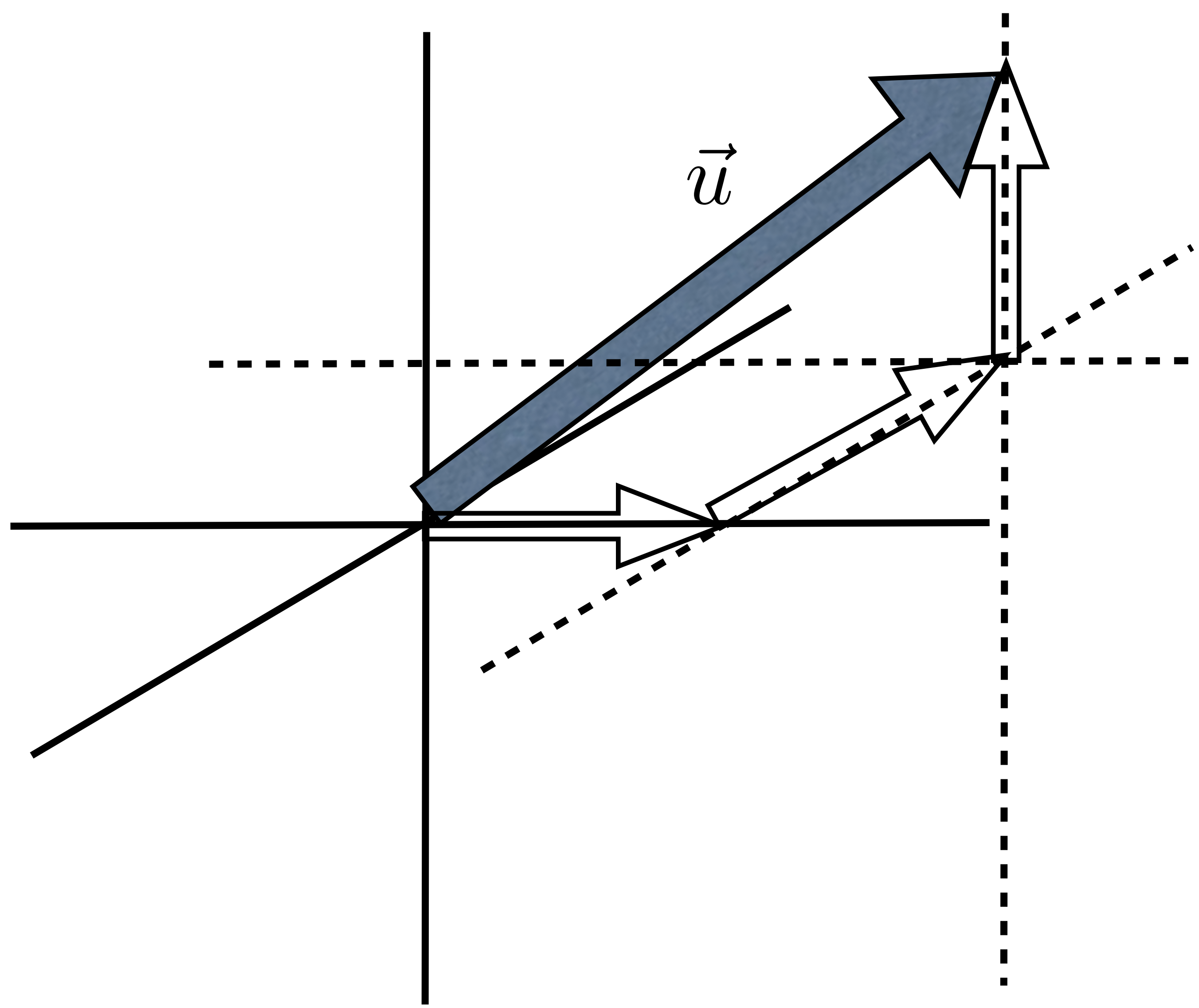
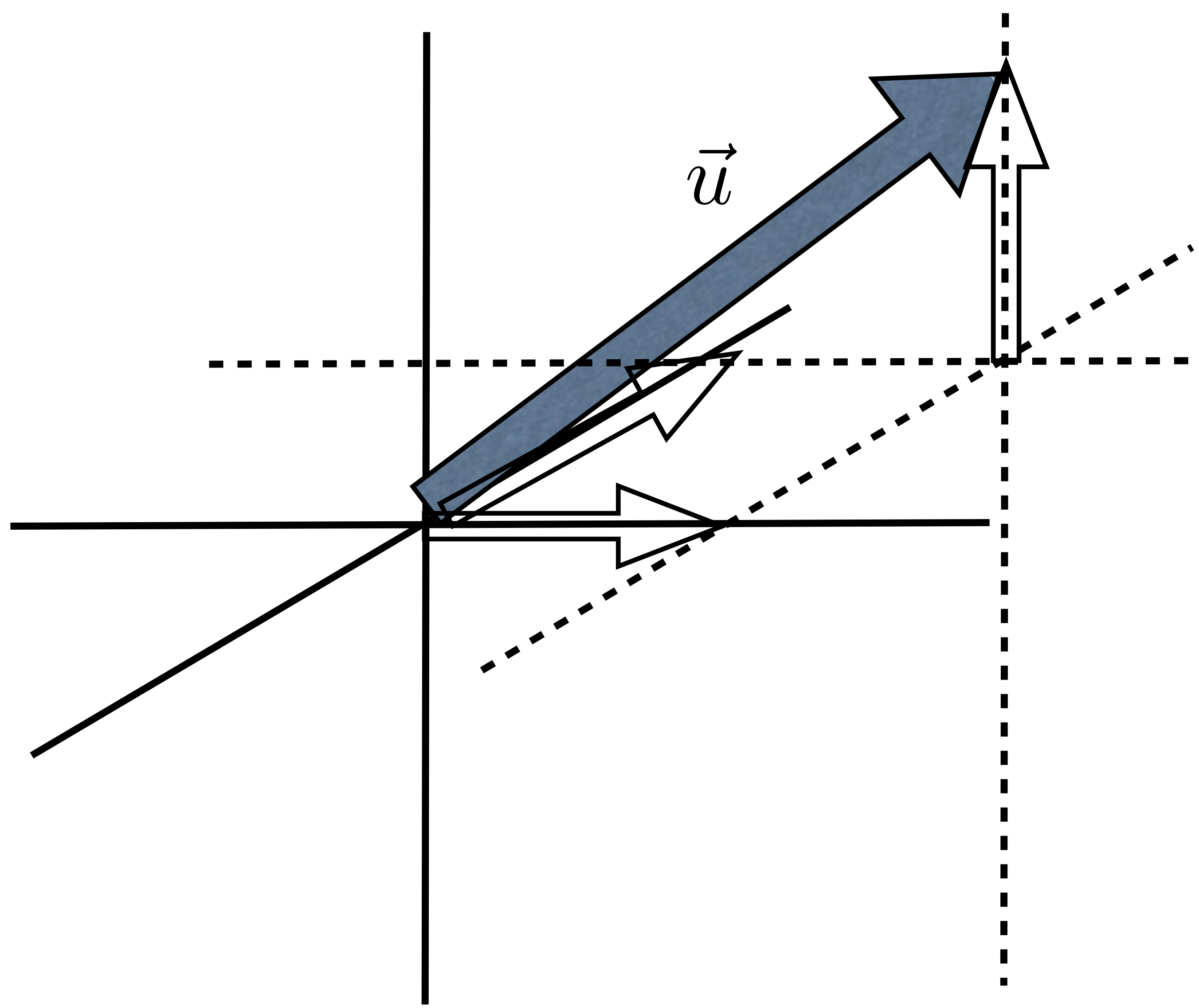


$\vec{u}$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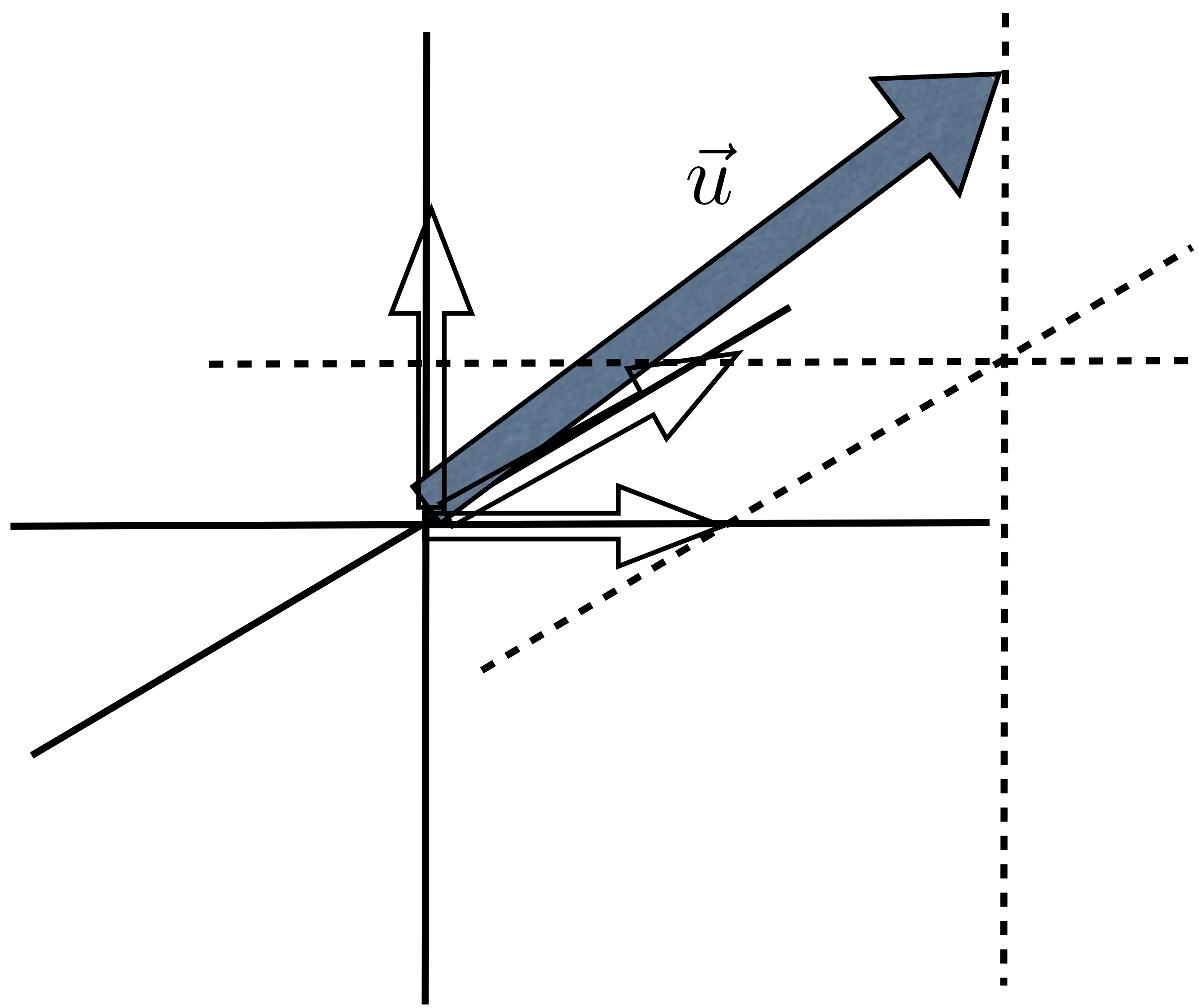
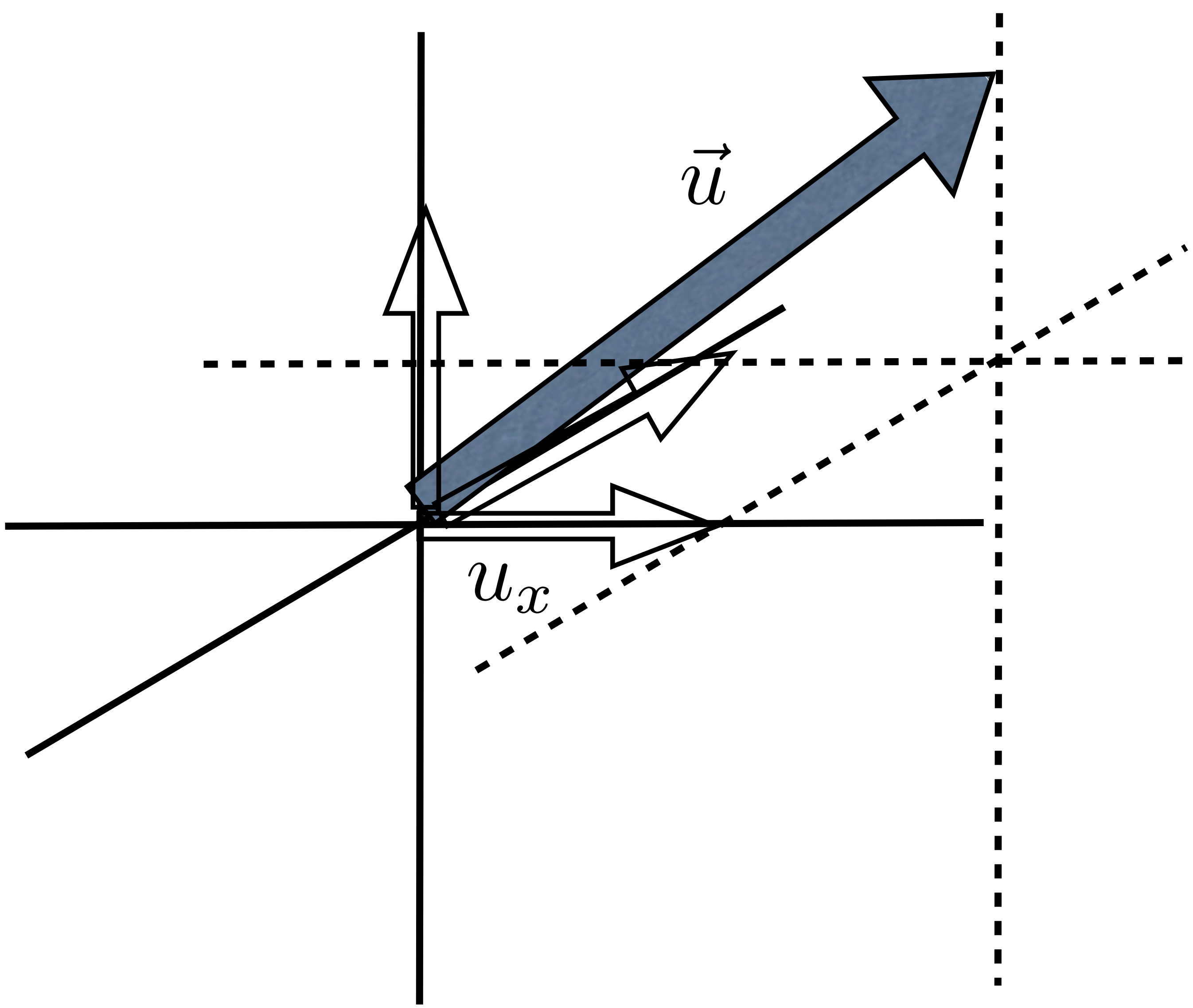
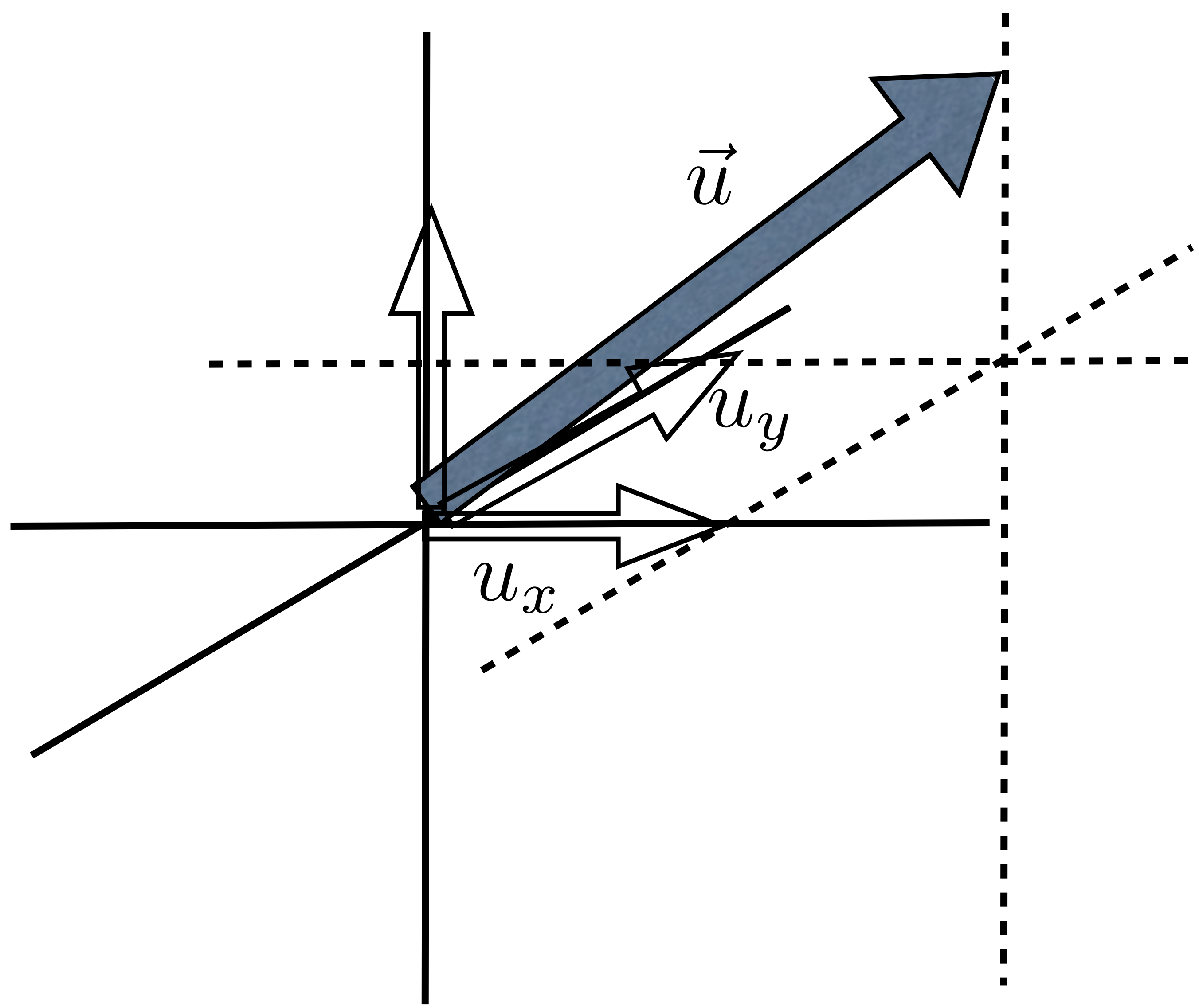
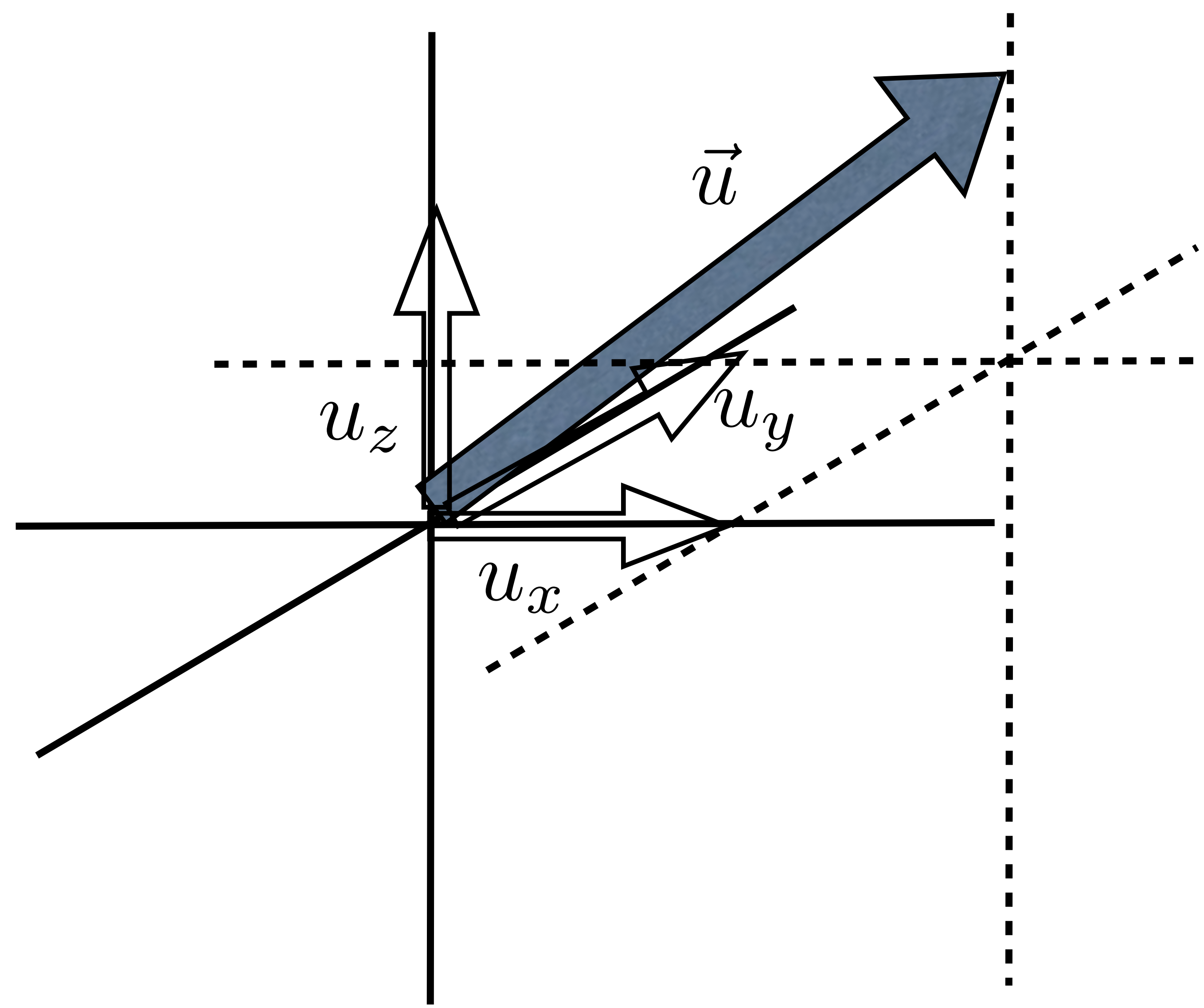

$\vec{u}$


$\vec{u}$
u_x

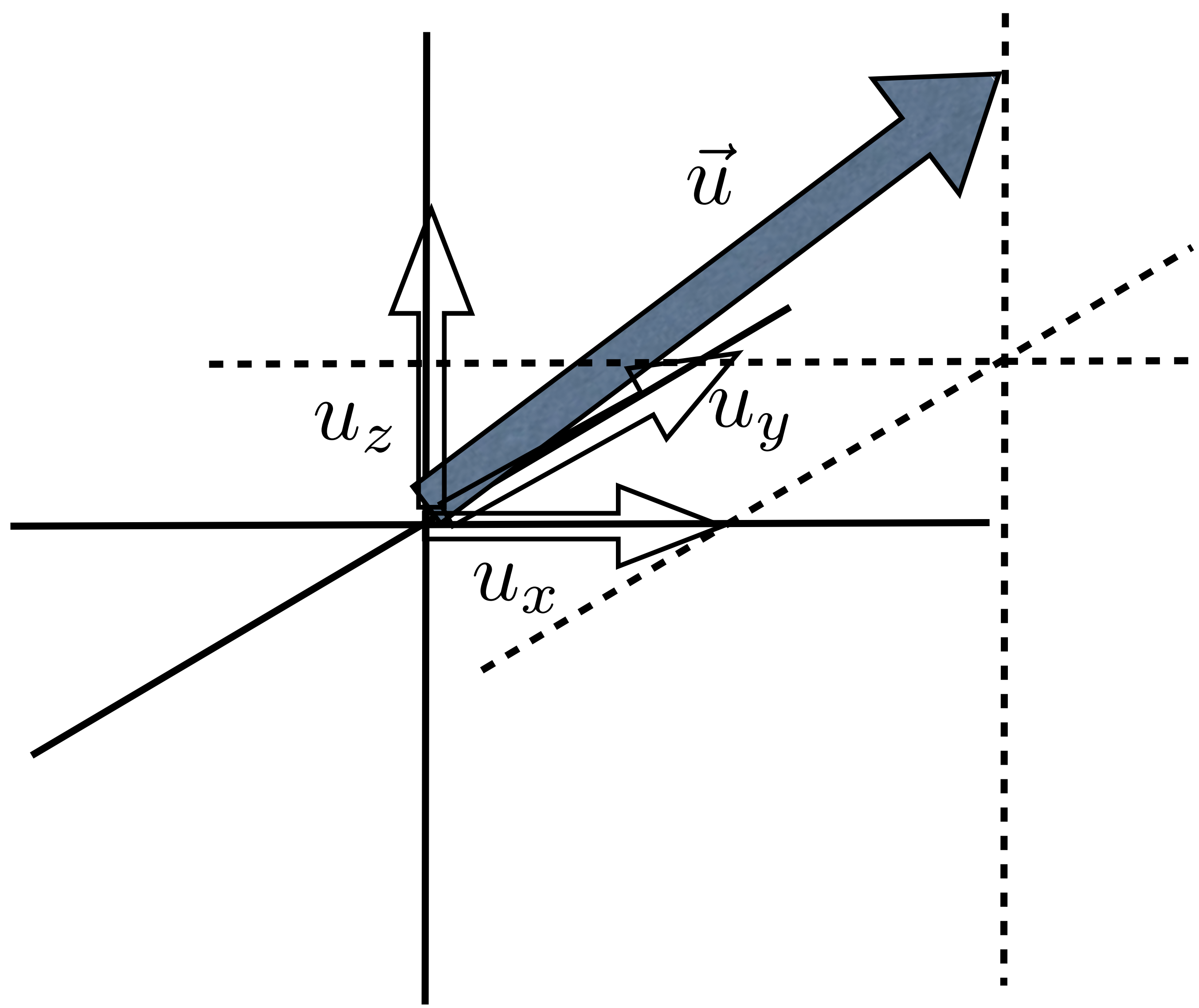

$\vec{u}$
u_y
u_x



$\vec{u}$
u_z
u_y
u_x

$$\vec{u} = u_x\mathbf{i} + u_y\mathbf{j} + u_z\mathbf{k}$$

$$|\vec{u}| = u$$



統計力学を使って
来年証明する

$$E_{\rm kinetic} = N\frac{3}{2}k_B T$$

$$\frac{1}{2}m\langle u^2\rangle = \frac{3}{2}k_B T$$

$$\frac{1}{2}m\langle u_x^2\rangle = \frac{1}{2}m\langle u_y^2\rangle = \frac{1}{2}m\langle u_z^2\rangle = \frac{1}{2}k_B T$$

$$\Longrightarrow \quad PV = Nk_BT = nN_{\rm avo}k_BT = nRT$$

$$\langle u^2 \rangle = \frac{3k_\mathrm{B}T}{m}$$

$$\sqrt{\langle u^2 \rangle} = \sqrt{\frac{3k_\mathrm{B}T}{m}}$$

N_2: 515 m s^{-1}

M&S のエレガントなMaxwell-Boltzmann分布の導入

$$h(u_x, u_y, u_z)du_x du_y du_z$$

$u_x \sim u_x + du_x$
$u_y \sim u_y + du_y$
$u_z \sim u_z + du_z$

の間に速度成分を持つ確率

*x,y,z*は独立 $h(u_x, u_y, u_z) = f(u_x)f(u_y)f(u_z)$

気体は等方的 $h(u) = h(u_x, u_y, u_z) = f(u_x)f(u_y)f(u_z)$

$$\mathbf{u} \cdot \mathbf{u} = u^2 = u_x^2 + u_y^2 + u_z^2$$

$$\ln h(u) = \ln f(u_x) + \ln f(u_y) + \ln f(u_z)$$

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気体は等方的

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$$\ln h(u) = \ln f(u_x) + \ln f(u_y) + \ln f(u_z)$$

$$\frac{\partial \ln h(u)}{\partial u_x} = \frac{d \ln f(u_x)}{d u_x} \qquad u = \left(u_x^2 + u_y^2 + u_z^2\right)^{1/2}$$

$$\frac{\partial \ln h(u)}{\partial u_x} = \frac{\partial u}{\partial u_x} \frac{d \ln h(u)}{du}$$

$$= (1/2)(u_x^2 + u_y^2 + u_z^2)^{-1/2}(2u_x)\frac{d \ln h(u)}{du}$$

$$= \frac{u_x}{u} \frac{d \ln h(u)}{du}$$

$$\frac{1}{u}\frac{d \ln h(u)}{du} = \frac{1}{u_x}\frac{d \ln f(u_x)}{du_x} = \frac{1}{u_y}\frac{d \ln f(u_y)}{du_y} = \frac{1}{u_z}\frac{d \ln f(u_z)}{du_z}$$

$$= -2\gamma$$

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$$\frac{1}{u}\frac{d \ln h(u)}{du} = \frac{1}{u_x}\frac{d \ln f(u_x)}{du_x} = \frac{1}{u_y}\frac{d \ln f(u_y)}{du_y} = \frac{1}{u_z}\frac{d \ln f(u_z)}{du_z}$$

$$= -2\gamma$$

$$\frac{d \ln f(u_x)}{du_x} = -2\gamma u_x$$

$$\ln f(u_x) = C - \gamma u_x^2$$

$$f(u_x) = Ae^{-\gamma u_x^2}$$

$$\int_{-\infty}^{\infty} f(u_x) du_x = 1$$

確率の総和は1

$$A \int_{-\infty}^{\infty} e^{-\gamma u_x^2} du_x = A\sqrt{\frac{\pi}{\gamma}} = 1,\ A = \sqrt{\frac{\gamma}{\pi}}$$

$$f(u_x) = \sqrt{\frac{\gamma}{\pi}} e^{-\gamma u_x^2}$$

$$\frac{d \ln f(u_x)}{du_x} = -2\gamma u_x$$

$$\ln f(u_x) = C - \gamma u_x^2$$

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確率の総和は1

$$A \int_{-\infty}^{\infty} e^{-\gamma u_x^2} du_x = A\sqrt{\frac{\pi}{\gamma}} = 1, \ A = \sqrt{\frac{\gamma}{\pi}}$$

$$f(u_x) = \sqrt{\frac{\gamma}{\pi}} e^{-\gamma u_x^2}$$

未知数γの決定

$$\langle u_x^2 \rangle = \frac{k_\mathrm{B}T}{m} = \frac{RT}{M}$$ を使う

$$\langle u_x^2 \rangle = \int_{-\infty}^{\infty} u_x^2 f(u_x) du_x = \sqrt{\frac{\gamma}{\pi}} \int_{-\infty}^{\infty} u_x^2 e^{-\gamma u_x^2} du_x$$

$$= \sqrt{\frac{\gamma}{\pi}} \frac{1}{2\gamma} \sqrt{\frac{\pi}{\gamma}} = \frac{1}{2\gamma}$$

$$\gamma = \frac{M}{2RT}$$

$$f(u_x) = \sqrt{\frac{M}{2\pi RT}} e^{-\frac{M}{2RT} u_x^2}$$

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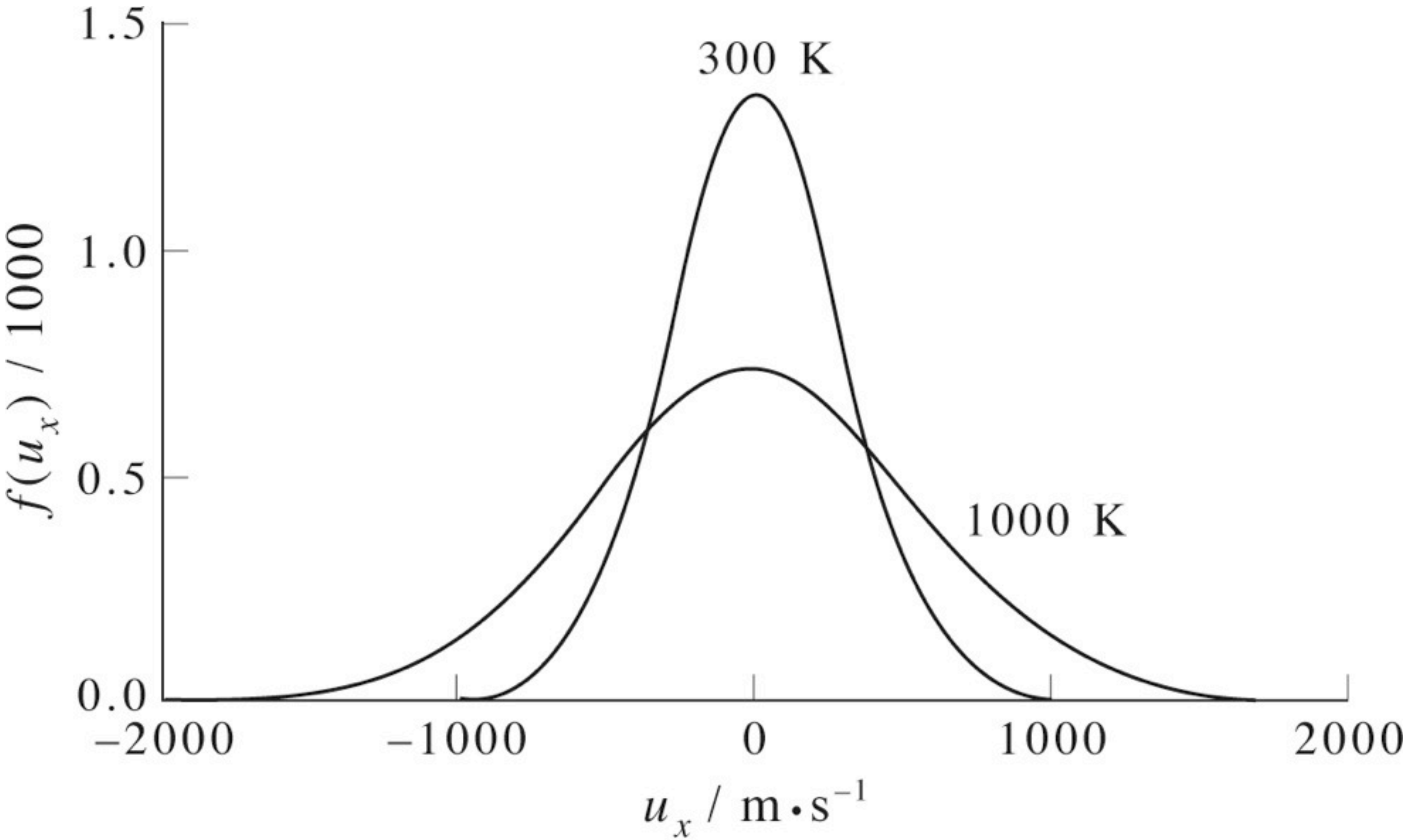
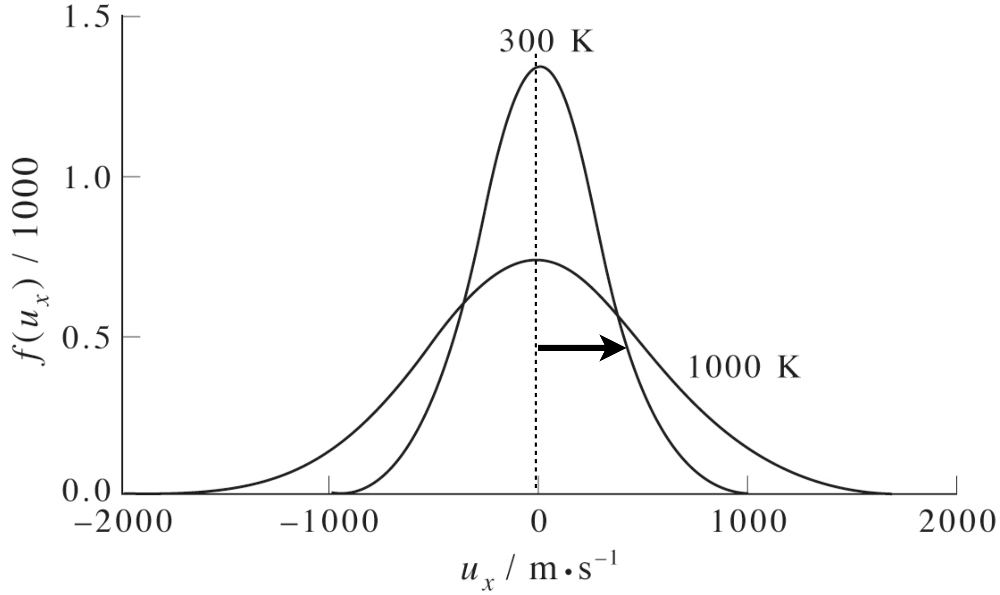


$$\langle u_x \rangle = \int_{-\infty}^{\infty} u_x f(u_x) du_x = 0$$



$$\langle u_{x,\rightarrow} \rangle = \int_0^\infty u_x f(u_x) \mathrm{d}u_x = \sqrt{\frac{M}{2\pi RT}} \int_0^\infty u_x e^{-\frac{M}{2RT}u_x^2} \mathrm{d}u_x$$

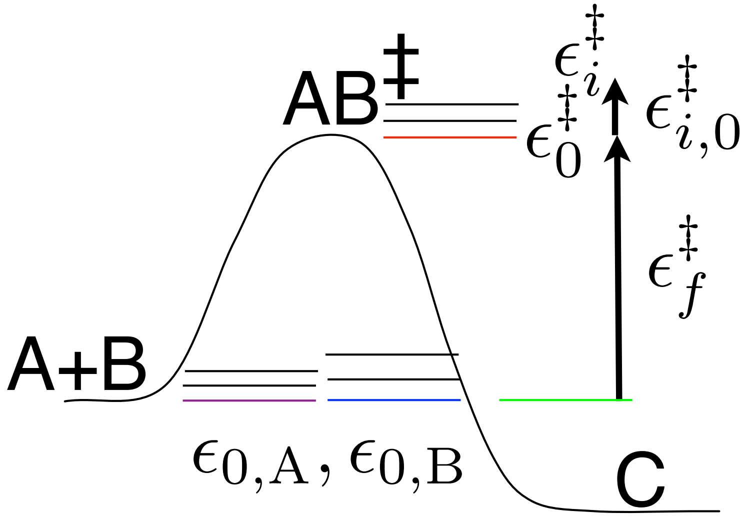
$$xe^{-ax^2} = \frac{-1}{2a}(e^{-ax^2})', \int_0^\infty xe^{-ax^2}\mathrm{d}x = \frac{-1}{2a}[e^{-ax^2}]_0^\infty = \frac{1}{2a}$$

$$\langle u_{x,\rightarrow} \rangle = \sqrt{\frac{M}{2\pi RT}} \frac{2RT}{2M} = \sqrt{\frac{RT}{2\pi M}}$$

$$
\begin{aligned}
q^{\ddagger}_{\mathrm{int,AB}} &= \sum_{i,\mathrm{int}} \exp\left(-\frac{\epsilon^{\ddagger}_{i}}{k_{\mathrm{B}}T}\right) \\
q_{\mathrm{A}} &= \sum_{i} \exp\left(-\frac{\epsilon_{i,A}}{k_{\mathrm{B}}T}\right), \quad q_{\mathrm{B}} = \sum_{i} \exp\left(-\frac{\epsilon_{i,B}}{k_{\mathrm{B}}T}\right) \\
\epsilon^{\ddagger}_{0} &= \epsilon_{0,A} + \epsilon^{\ddagger}_{f} = \epsilon_{0,B} + \epsilon^{\ddagger}_{f} \\
q^{\ddagger}_{\mathrm{int,AB}} &= \sum_{i,\mathrm{int}} \exp\left(-\frac{\epsilon^{\ddagger}_{i,0}}{k_{\mathrm{B}}T}\right) \exp\left(-\frac{\epsilon^{\ddagger}_{f}}{k_{\mathrm{B}}T}\right) \\
&= q^{\ddagger}_{\mathrm{int,0,AB}} \exp\left(-\frac{\epsilon^{\ddagger}_{f}}{k_{\mathrm{B}}T}\right)
\end{aligned}
$$

$$\epsilon^{\ddagger}_{i} = \epsilon^{\ddagger}_{i,0} + \epsilon^{\ddagger}_{f}$$

→エネルギーゼロ点の差



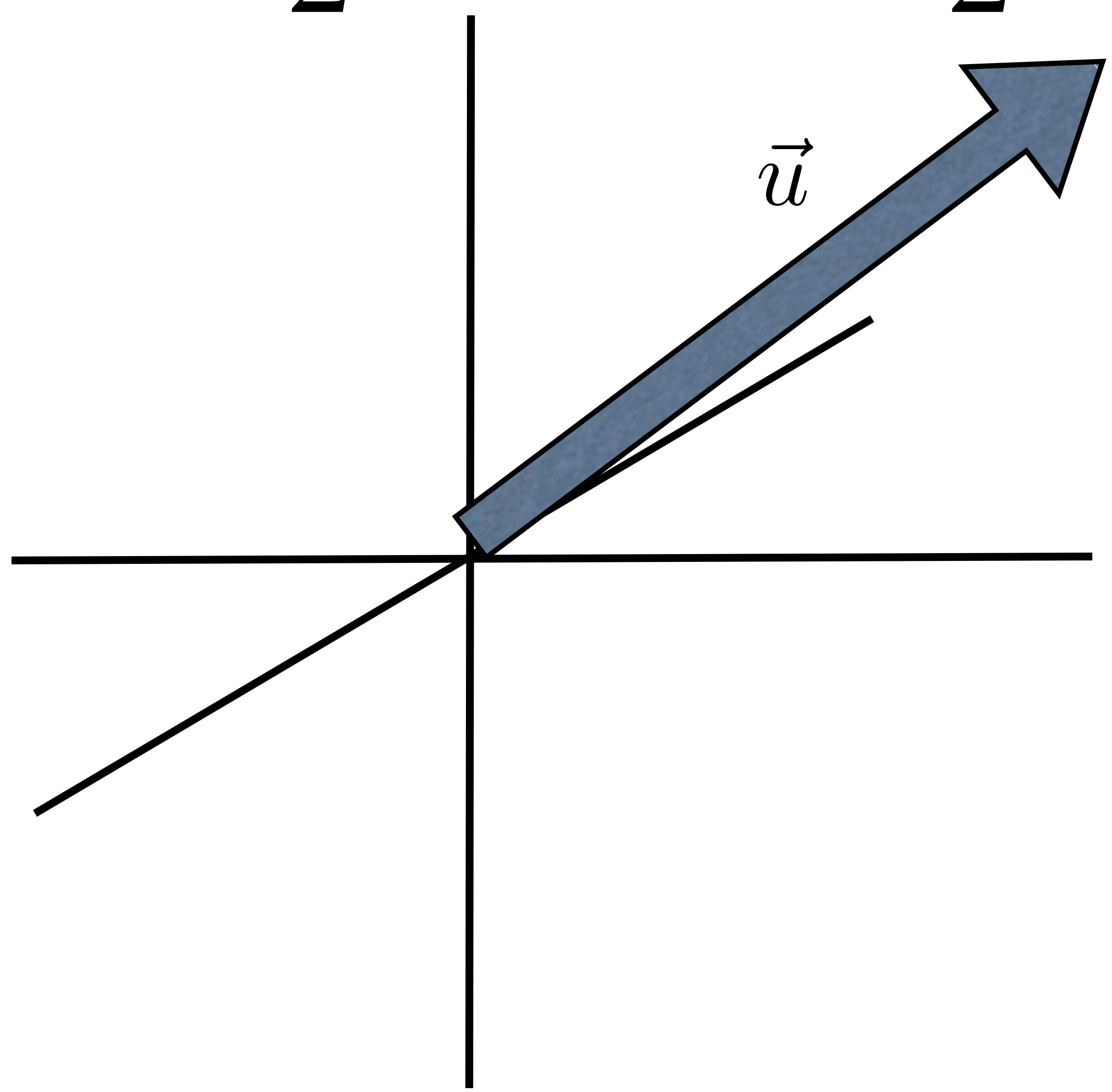
$$
\begin{aligned}
\nu\delta &= \langle v_{\rightarrow} \rangle \\
\langle v_{\rightarrow} \rangle &= \int_{0}^{\infty} v_x f(v_x) dv_x \qquad \textbf{(28.68)}
\end{aligned}
$$

$$
\begin{aligned}
f(v_x) &= \sqrt{\frac{m^{\ddagger}}{2\pi k_{\mathrm{B}}T}} \exp\left(-\frac{m^{\ddagger} v_x^2}{2k_{\mathrm{B}}T}\right), \quad \int_{-\infty}^{\infty} f(v_x) dv_x = 1 \\
\langle v_{\rightarrow} \rangle &= \sqrt{\frac{m^{\ddagger}}{2\pi k_{\mathrm{B}}T}} \int_{0}^{\infty} v_x \exp\left(-\frac{m^{\ddagger} v_x^2}{2k_{\mathrm{B}}T}\right) dv_x = \sqrt{\frac{m^{\ddagger}}{2\pi k_{\mathrm{B}}T}} \frac{2k_{\mathrm{B}}T}{2m^{\ddagger}} = \sqrt{\frac{k_{\mathrm{B}}T}{2\pi m^{\ddagger}}}
\end{aligned}
$$

$$u^2 = u_x^2 + u_y^2 + u_z^2$$

$$\langle u^2 \rangle = \langle u_x^2 + u_y^2 + u_z^2 \rangle = 3\langle u_x^2 \rangle$$

$$\frac{1}{2} m \langle u^2 \rangle = \frac{3}{2} m \langle u_x^2 \rangle = \frac{3}{2} k_{\mathrm{B}} T$$



$$u^2 = u_x^2 + u_y^2 + u_z^2$$

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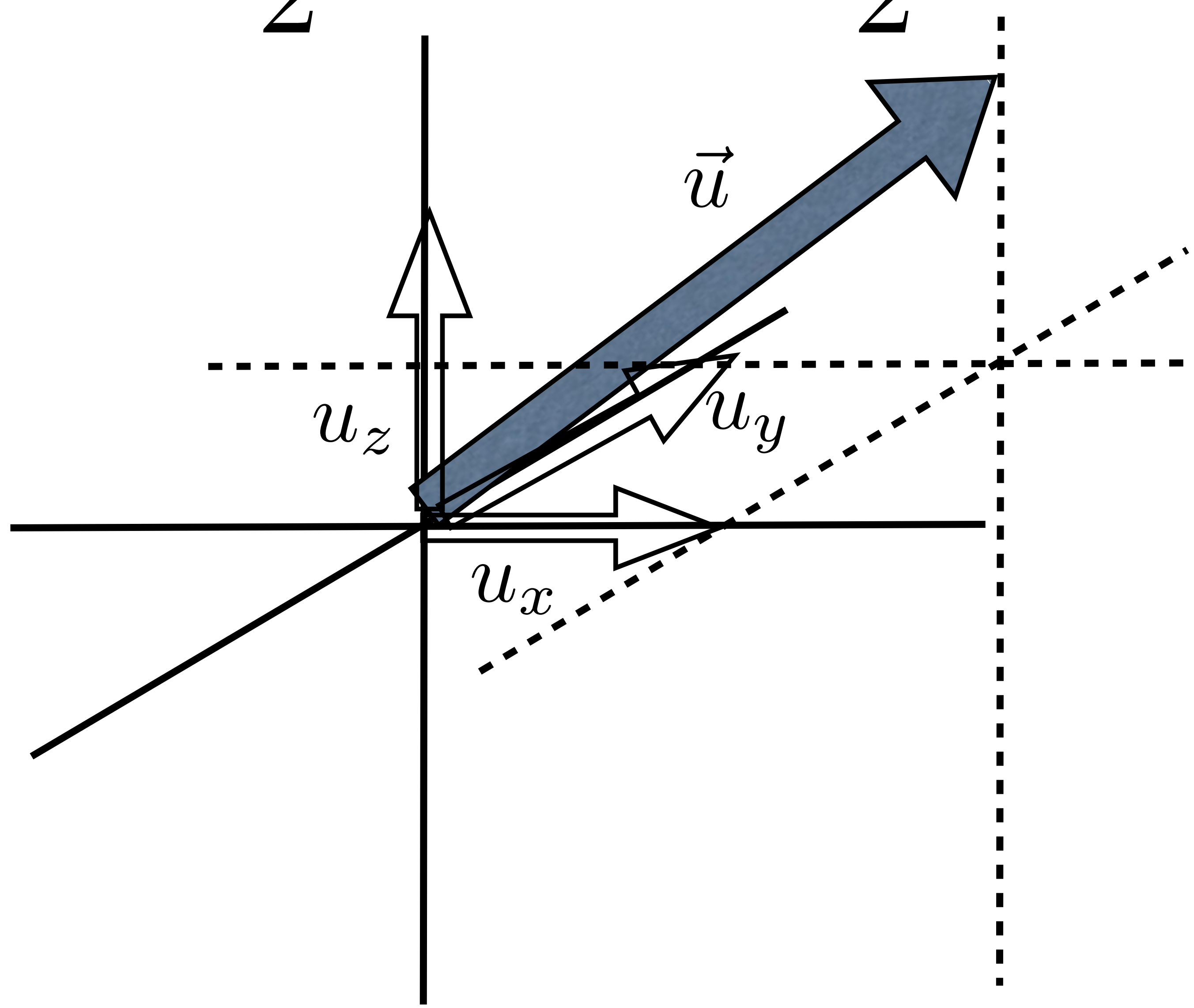
$$\frac{1}{2}m\langle u^2 \rangle = \frac{3}{2}m\langle u_x^2 \rangle = \frac{3}{2}k_{\mathrm{B}}T$$



$$I = \int_{-\infty}^{+\infty} e^{-ax^2} dx, \quad I^2 = \int_{-\infty}^{+\infty} e^{-ax^2} dx \int_{-\infty}^{+\infty} e^{-ay^2} dy$$

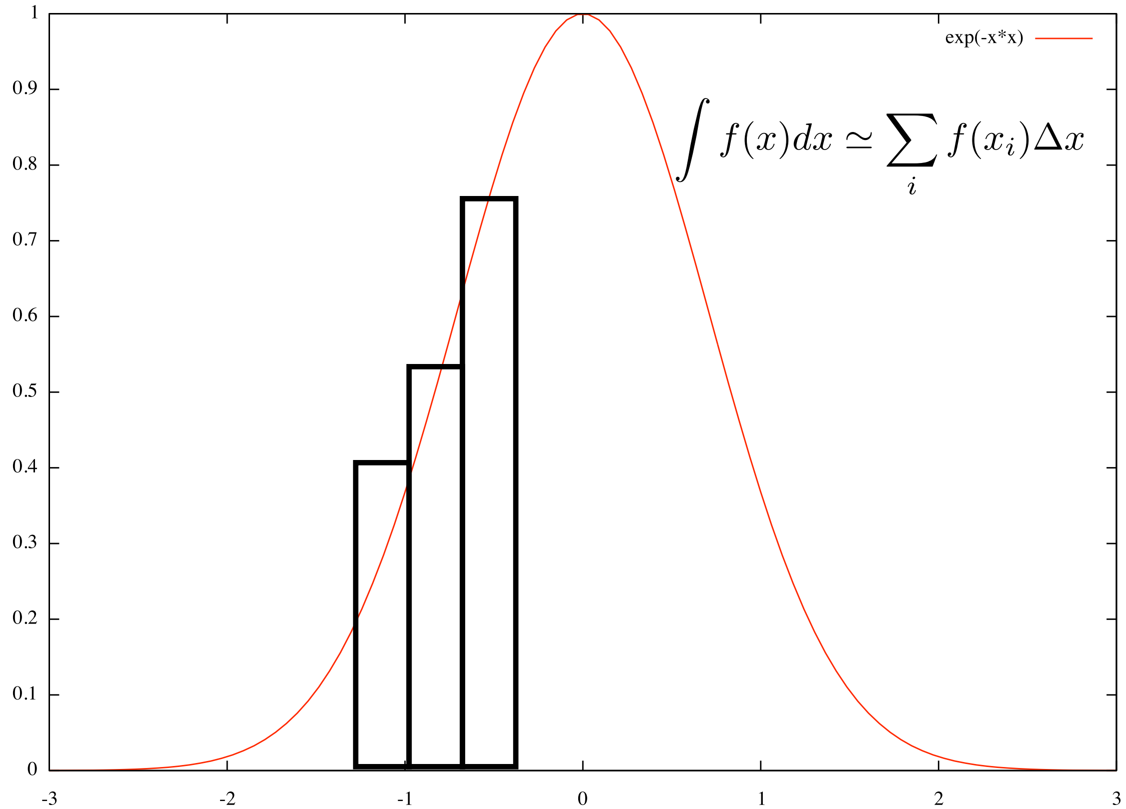
$$I^2 = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx dy e^{-a(x^2+y^2)} = \int_0^{\infty} r dr \int_0^{2\pi} d\theta e^{-ar^2}$$

$$R = r^2, \quad dR = 2r dr$$

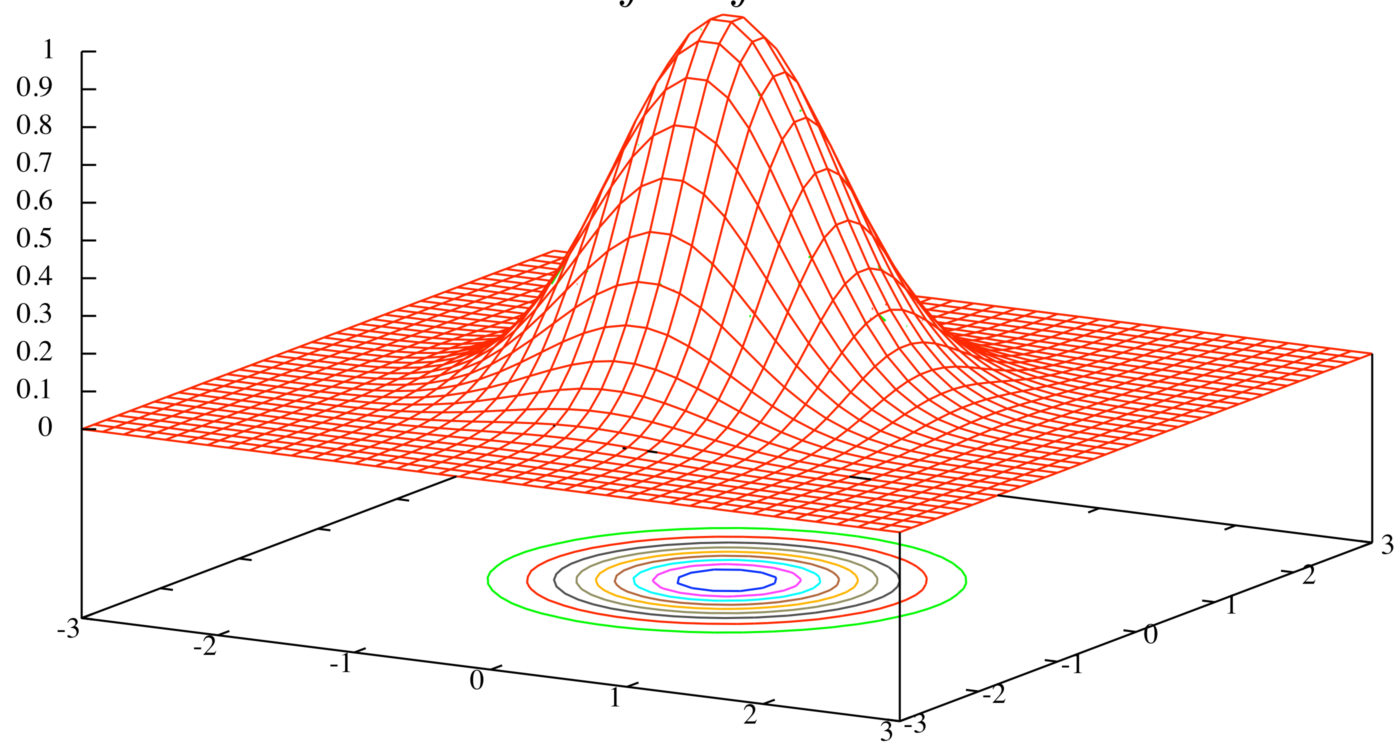
$$I^2 = \frac{1}{2} \int_0^{\infty} dR \int_0^{2\pi} d\theta e^{-aR} = \pi \left[-\frac{1}{a} e^{-aR} \right]_0^{\infty} = \frac{\pi}{a}$$

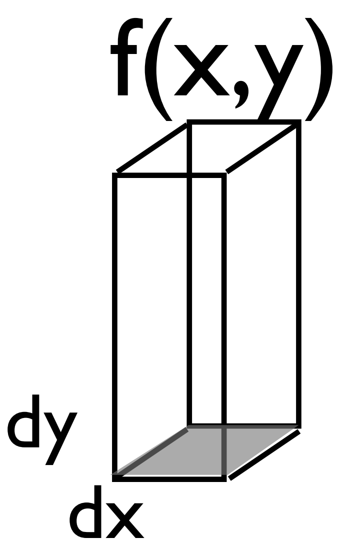
$$I = \sqrt{\frac{\pi}{a}}$$

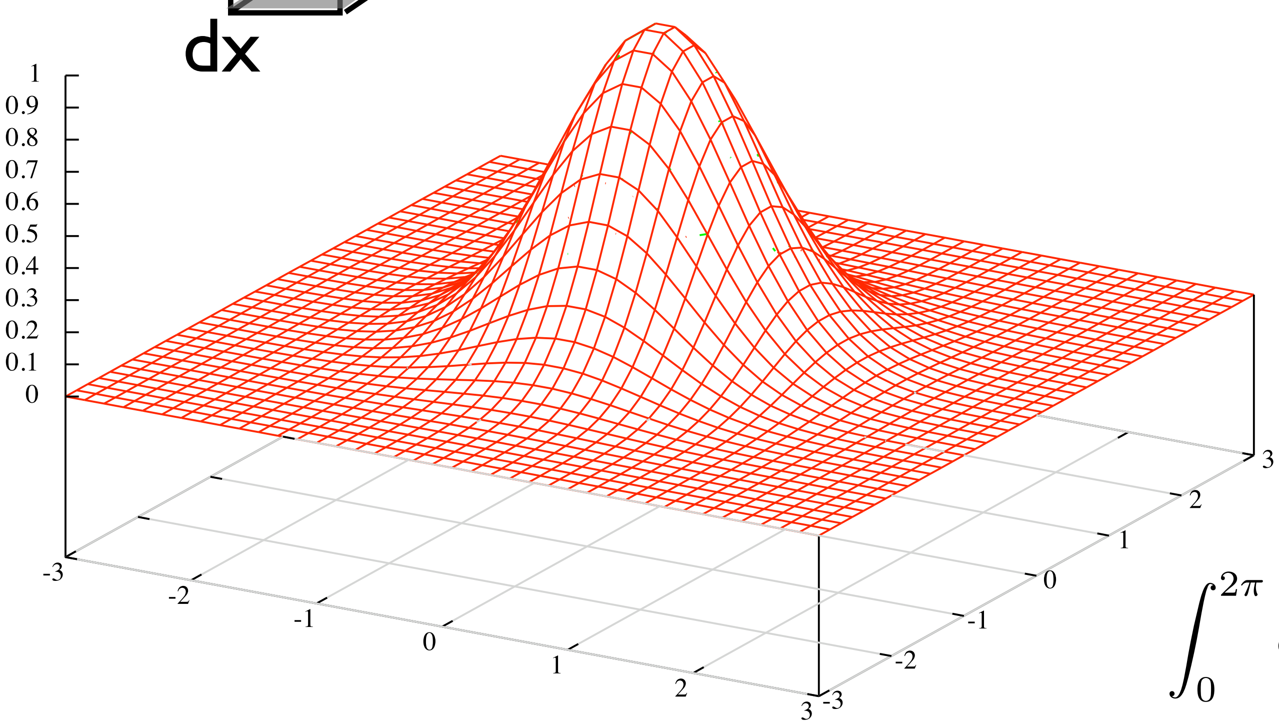
$$
\begin{aligned}
I &= \int_{-\infty}^{+\infty} e^{-ax^2} dx, \quad I^2 = \int_{-\infty}^{+\infty} e^{-ax^2} dx \int_{-\infty}^{+\infty} e^{-ay^2} dy \\
I^2 &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dxdye^{-a(x^2+y^2)} = \int_0^{\infty} rdr \int_0^{2\pi} d\theta e^{-ar^2} \\
R &= r^2, \quad dR = 2rdr \\
I^2 &= \frac{1}{2} \int_0^{\infty} dR \int_0^{2\pi} d\theta e^{-aR} = \pi \left[-\frac{1}{a} e^{-aR} \right]_0^{\infty} = \frac{\pi}{a} \\
I &= \sqrt{\frac{\pi}{a}}
\end{aligned}
$$



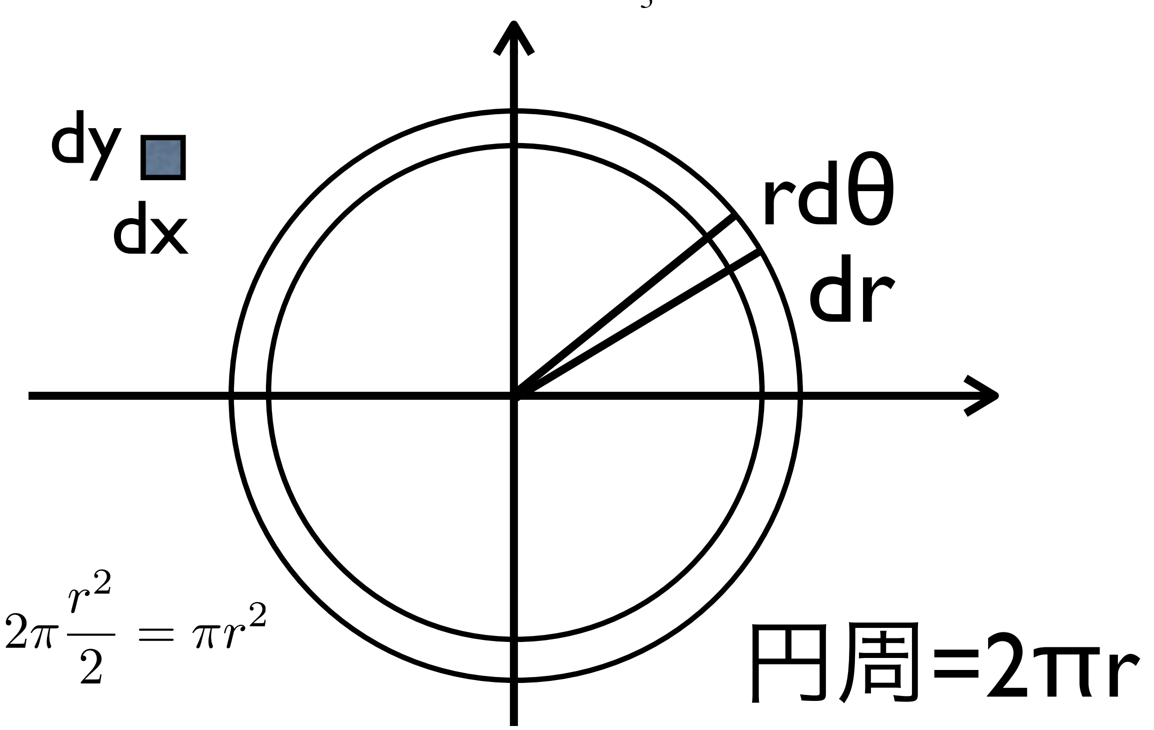
$$
\begin{aligned}
\int\int f(x,y)dxdy &\simeq \sum_i\sum_j f(x_i,y_j)\Delta x\Delta y \\
&= \sum_i\sum_j f(x_i,y_j)\Delta S \\
&= \sum_i\sum_j f(r_i,\theta_j)\Delta S \\
&= \int d\theta \int dr r f(r,\theta)
\end{aligned}
$$

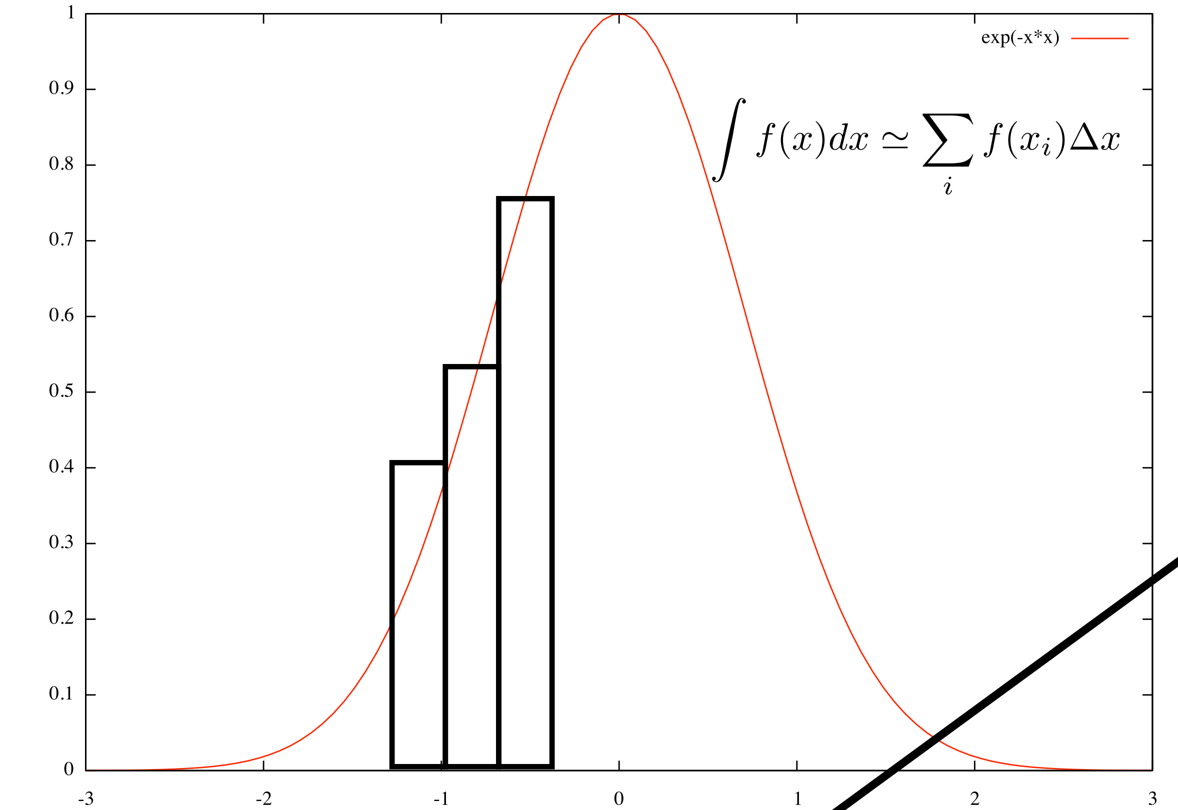




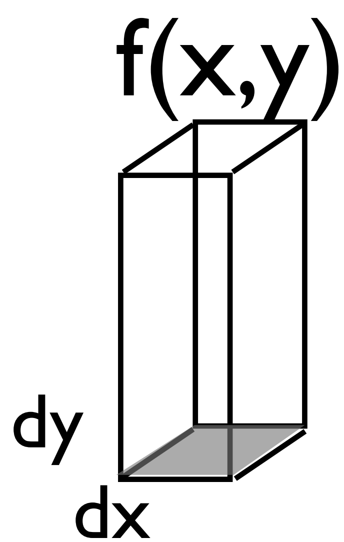


$$\int_0^{2\pi} d\theta \int_0^r r dr = 2\pi\frac{r^2}{2} = \pi r^2$$

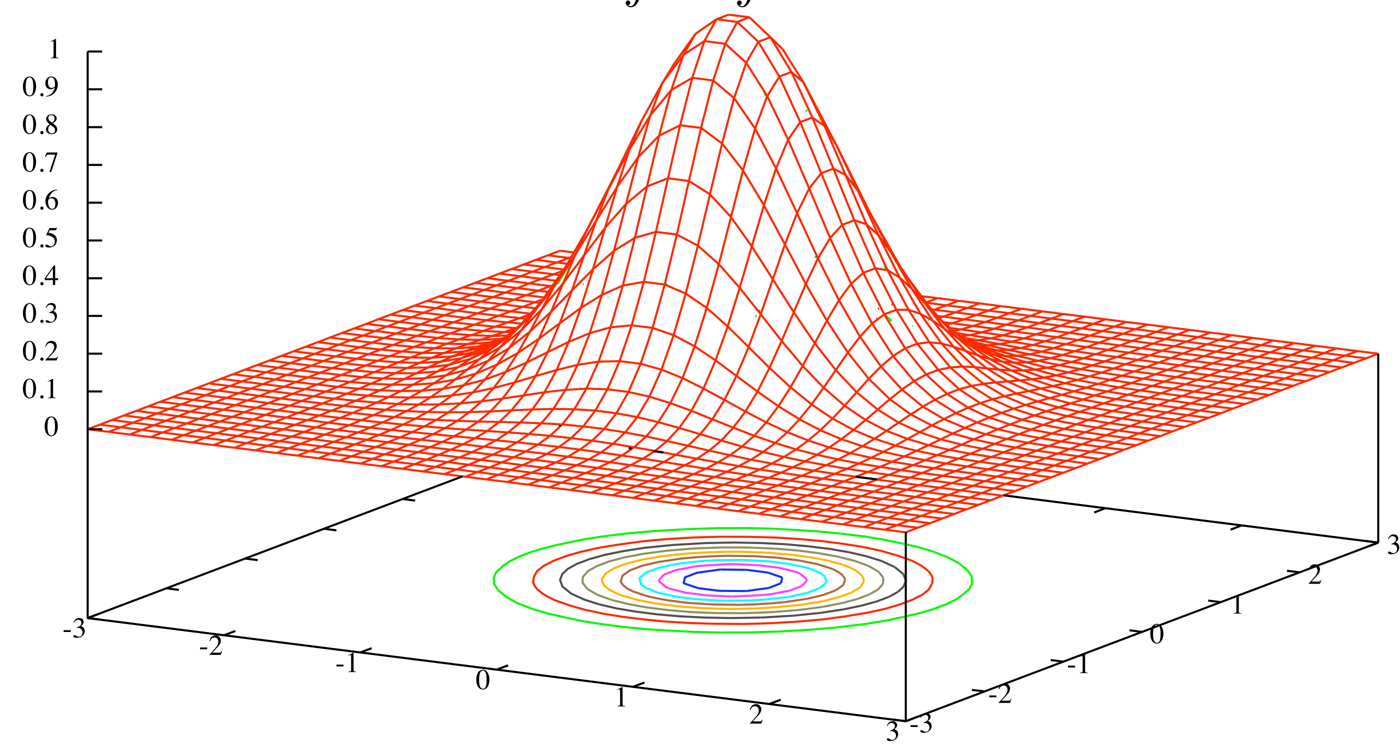




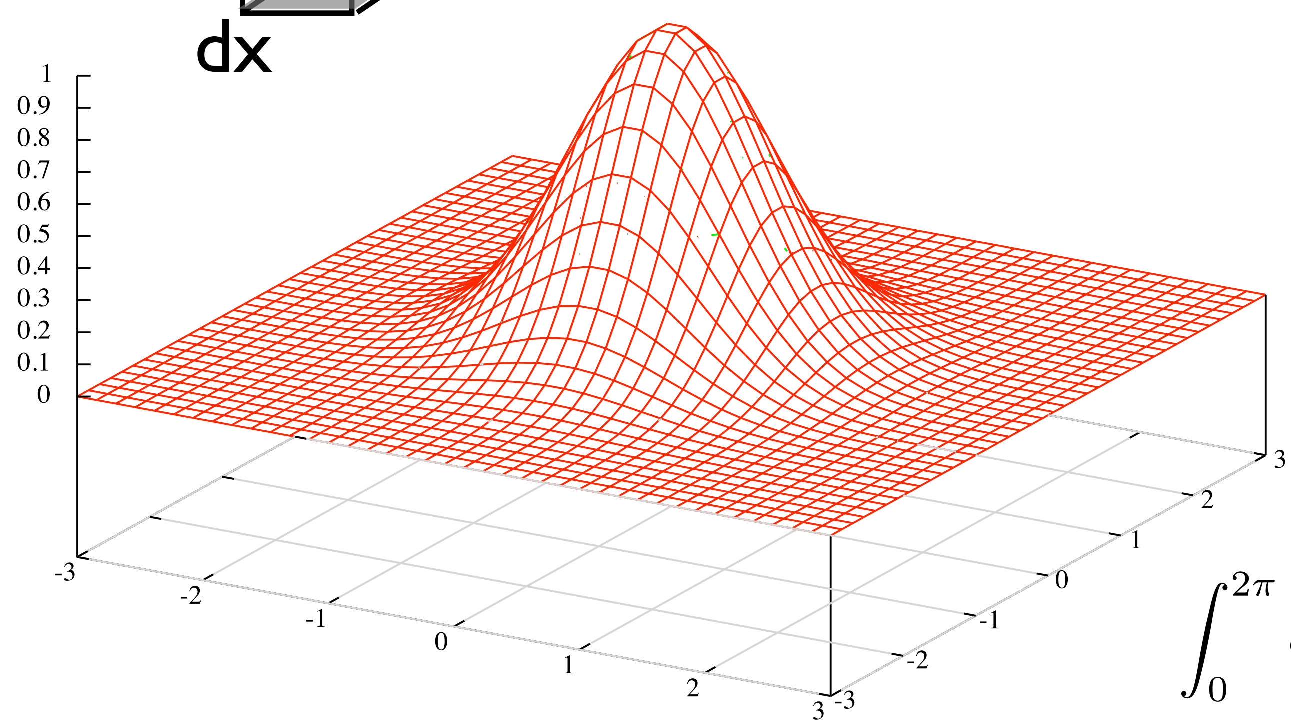
$$\begin{aligned}\int\int f(x,y)dxdy &\simeq \sum_i\sum_j f(x_i,y_j)\Delta x\Delta y\\ &= \sum_i\sum_j f(x_i,y_j)\Delta S\\ &= \sum_i\sum_j f(r_i,\theta_j)\Delta S\\ &= \int d\theta\int dr r f(r,\theta)\end{aligned}$$



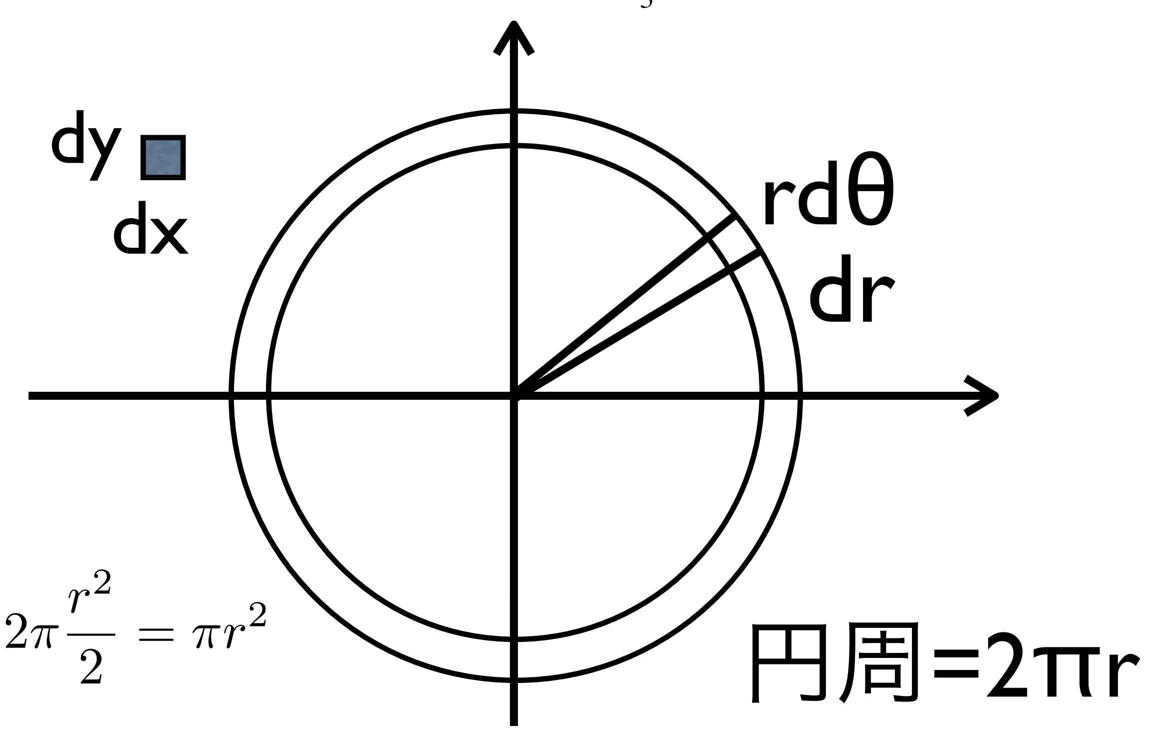
exp(-x*x)*exp(-y

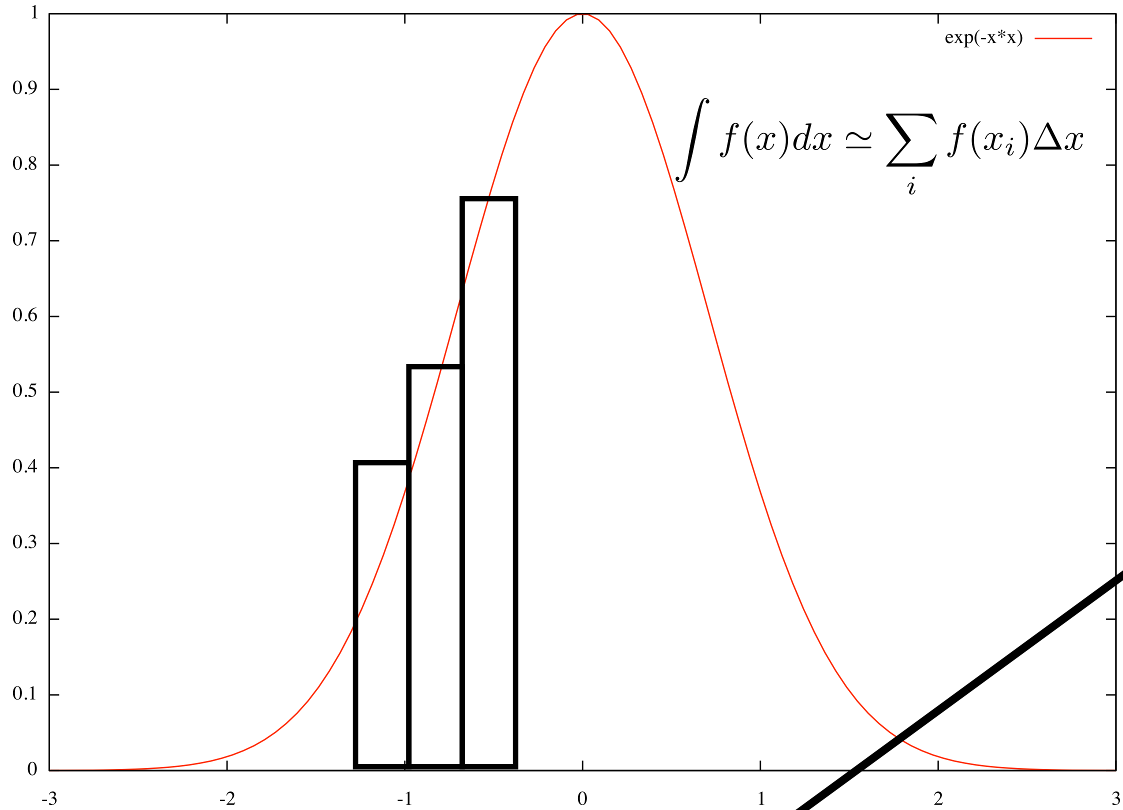


exp(-x*:



$$\int_0^{2\pi} d\theta \int_0^r rdr = 2\pi\frac{r^2}{2} = \pi r^2$$





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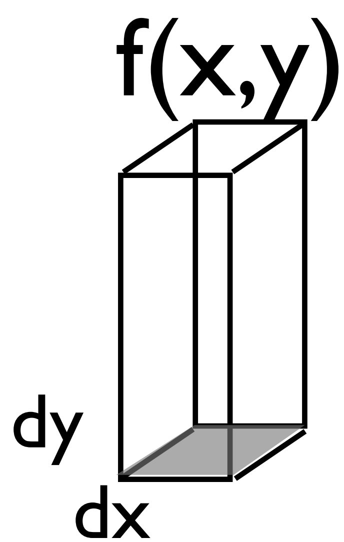
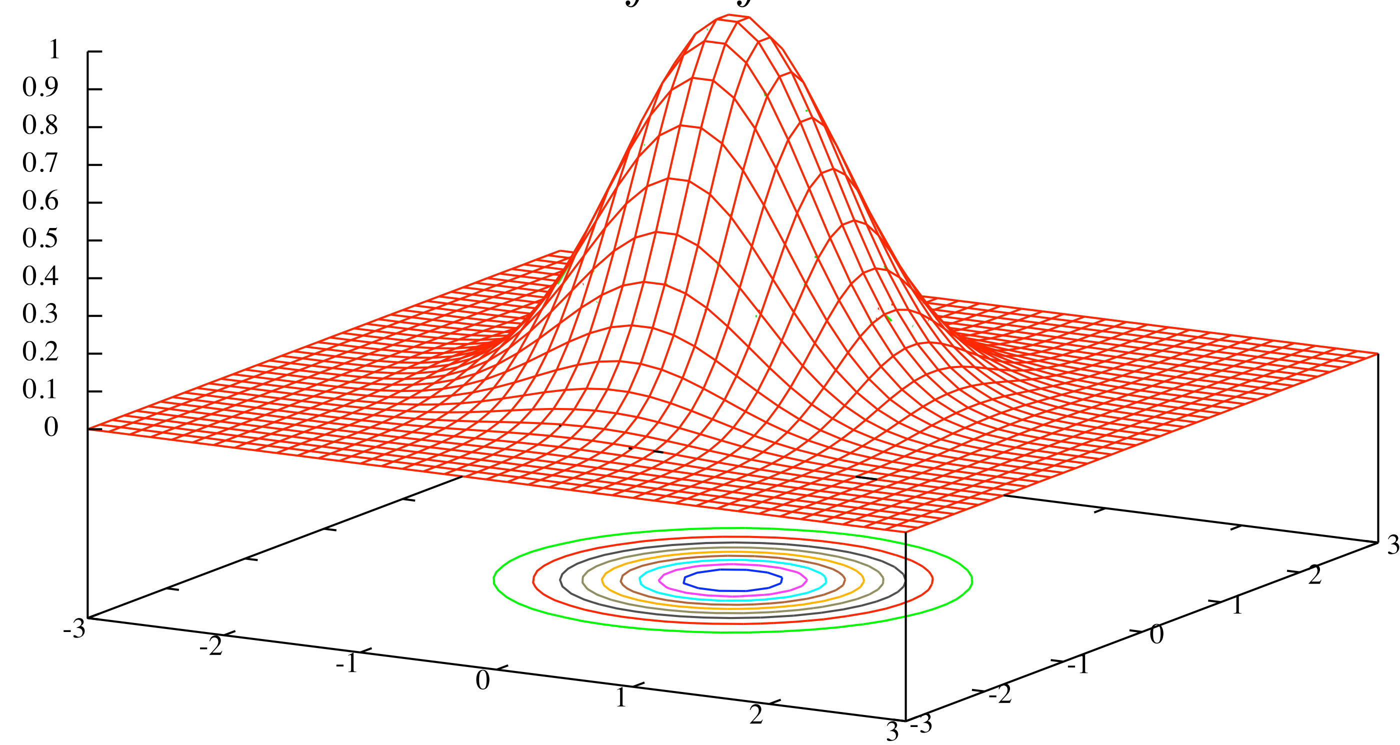
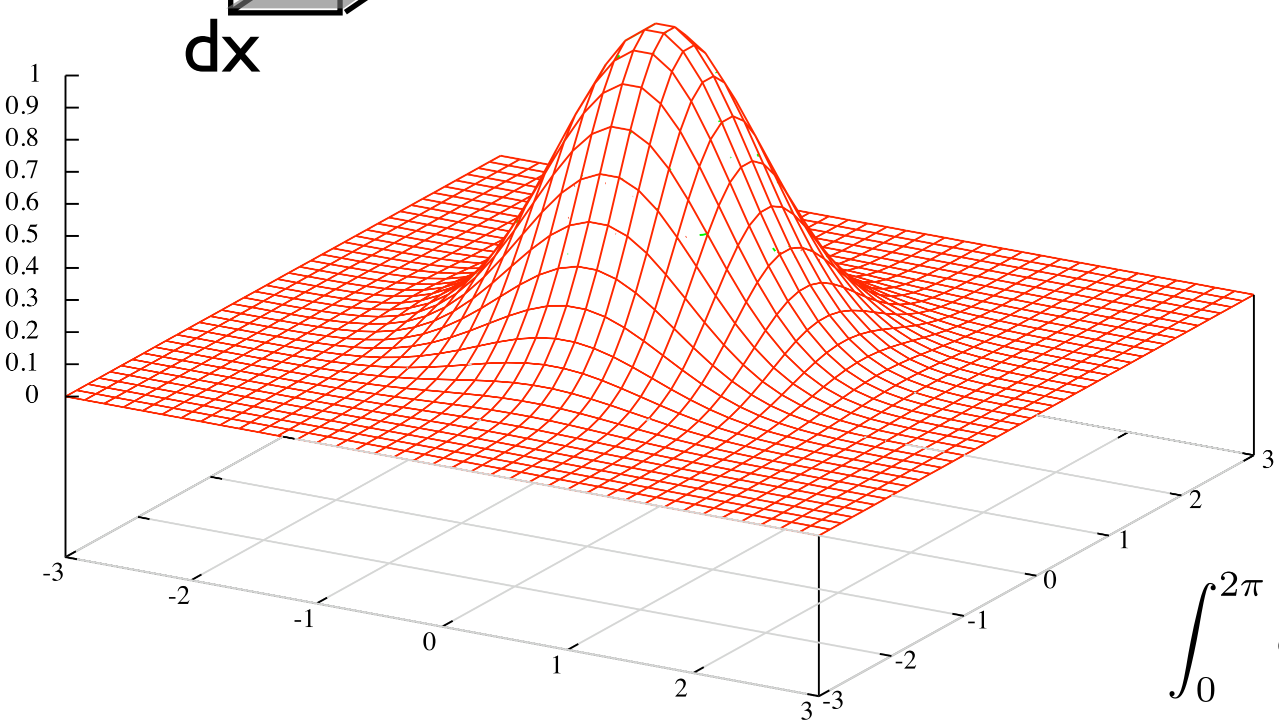
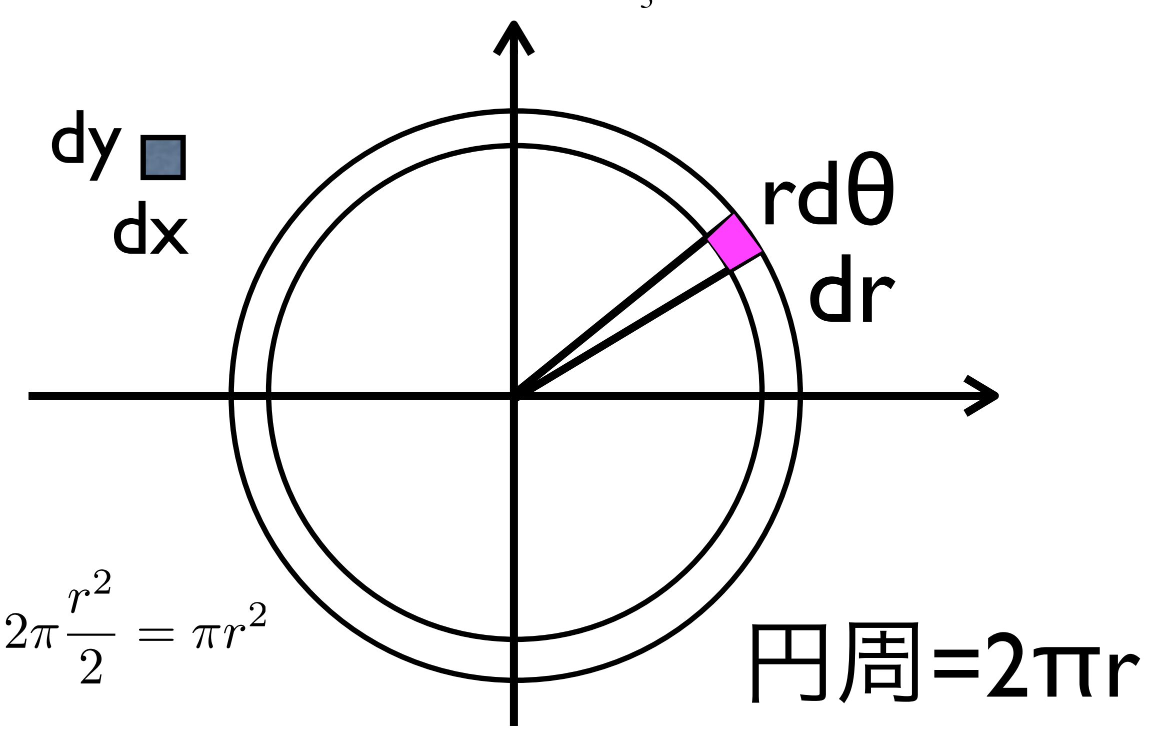


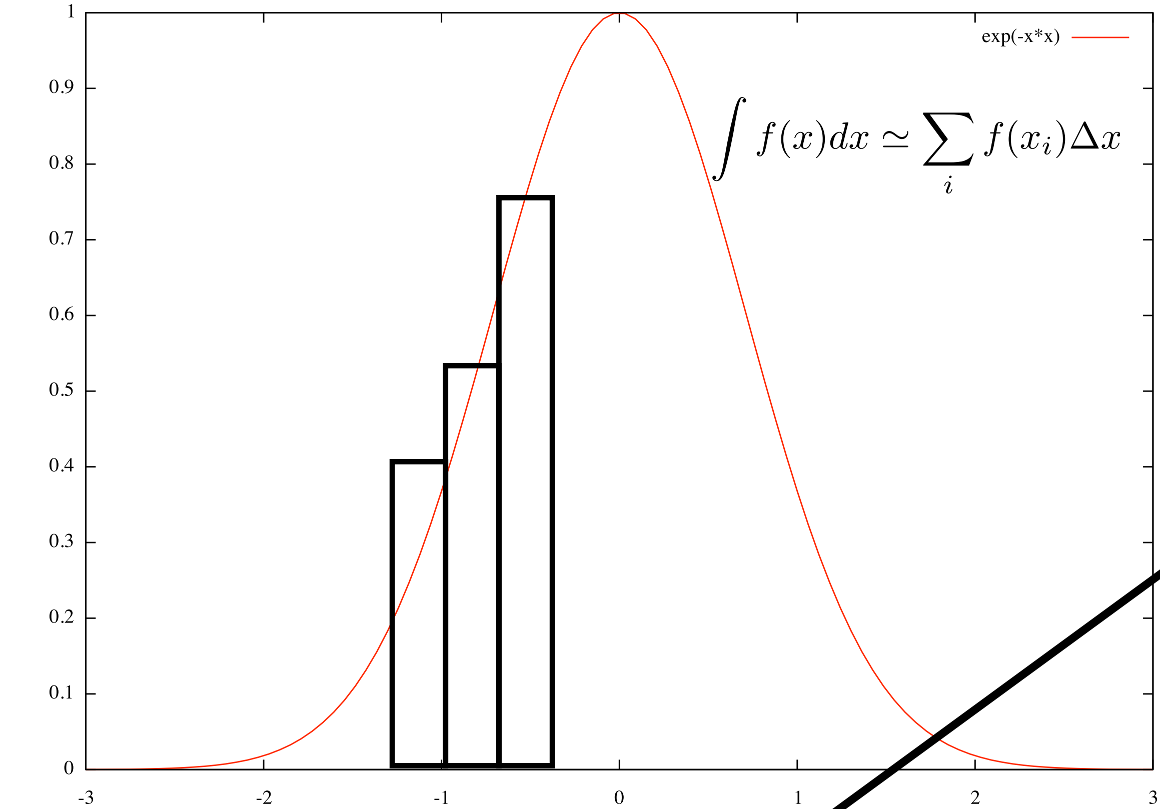


exp(-x*:



$$\int_0^{2\pi} d\theta \int_0^r r dr = 2\pi\frac{r^2}{2} = \pi r^2$$





$$
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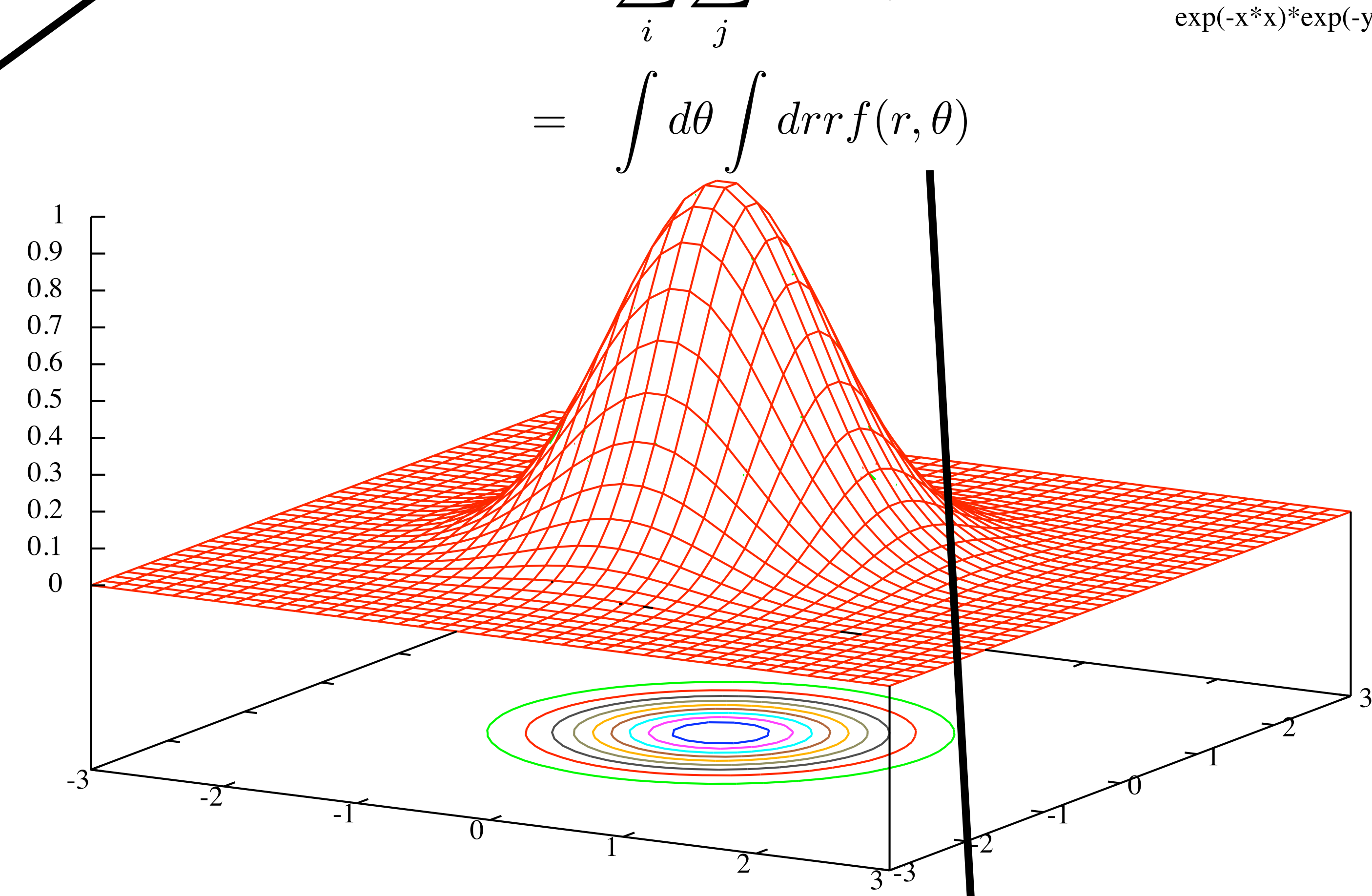
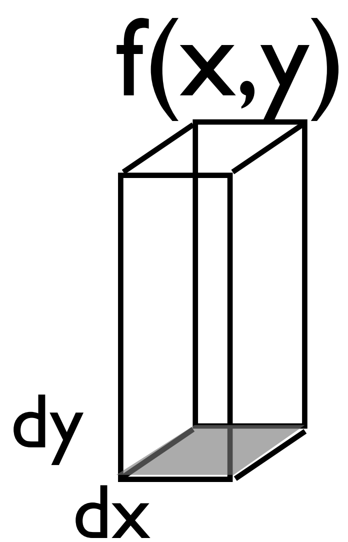
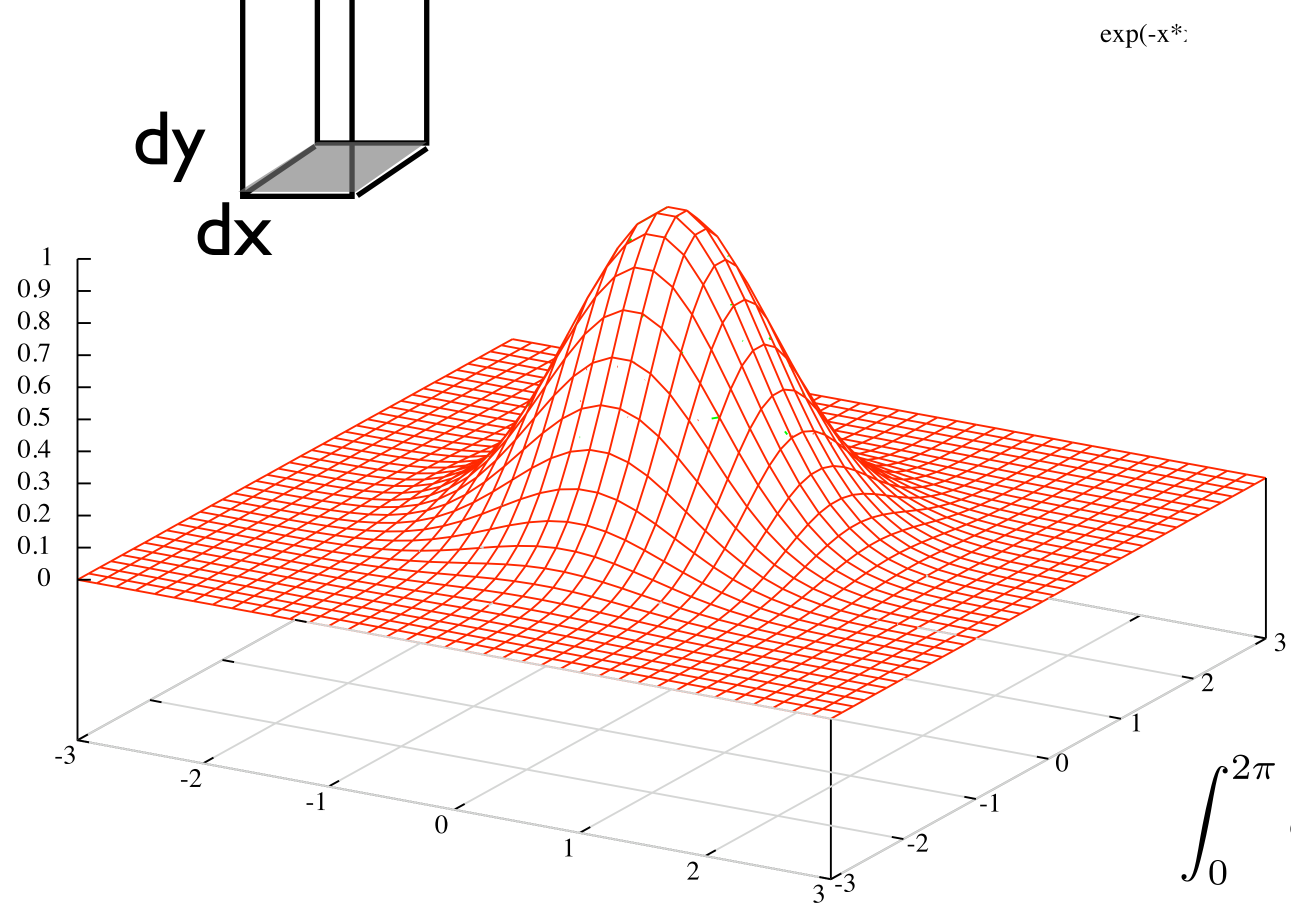
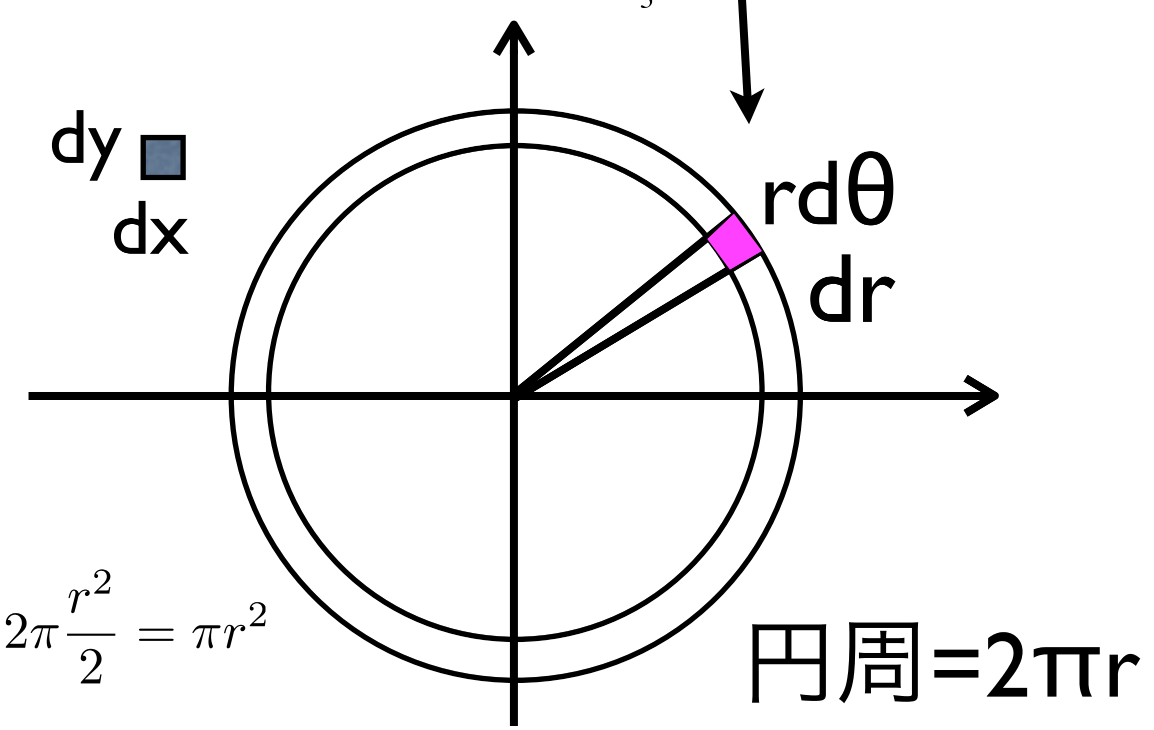
$$







$$\int_0^{2\pi} d\theta \int_0^r r dr = 2\pi\frac{r^2}{2} = \pi r^2$$



$$xe^{-ax^2} = \frac{-1}{2a}(e^{-ax^2})'$$

$$(fg)' = f'g + fg', \qquad \int f'g = fg - \int fg'$$

$$\int_0^\infty dx\, xe^{-ax^2} = \frac{-1}{2a}[e^{-ax^2}]_0^\infty = \frac{1}{2a}$$

$$\int_{-\infty}^\infty dx\, xe^{-ax^2} = 0, \qquad \int_{-\infty}^\infty \text{odd} \times \text{even} = 0$$

$$\int_{-\infty}^\infty dx\, x^2 e^{-ax^2} = \int_{-\infty}^\infty dx\, x \frac{-1}{2a}(e^{-ax^2})'$$

$$= \frac{-1}{2a}\left[e^{-ax^2}x\right]_{-\infty}^\infty + \frac{1}{2a}\int_{-\infty}^\infty dx\, e^{-ax^2}x' = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty dx\, x^2 e^{-ax^2} = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty dx\, x^3 e^{-ax^2} = 0$$

$$xe^{-ax^2} = \frac{-1}{2a}(e^{-ax^2})'$$

$$(fg)' = f'g + fg', \qquad \int f'g = fg - \int fg'$$

$$\int_0^\infty dx x e^{-ax^2} = \frac{-1}{2a}[e^{-ax^2}]_0^\infty = \frac{1}{2a}$$

$$\int_{-\infty}^\infty dx x e^{-ax^2} = 0, \qquad \int_{-\infty}^\infty \text{odd} \times \text{even} = 0$$

$$\int_{-\infty}^\infty dx x^2 e^{-ax^2} = \int_{-\infty}^\infty dx x \frac{-1}{2a}(e^{-ax^2})'$$

$$= \frac{-1}{2a}\left[e^{-ax^2}x\right]_{-\infty}^\infty + \frac{1}{2a}\int_{-\infty}^\infty dx e^{-ax^2}x' = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty dx x^2 e^{-ax^2} = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^\infty dx x^3 e^{-ax^2} = 0$$

$$
\begin{aligned}
xe^{-ax^2} &= \frac{-1}{2a}(e^{-ax^2})' \\
(fg)' &= f'g + fg', \qquad \int f'g = fg - \int fg' \\
\int_0^\infty dx x e^{-ax^2} &= \frac{-1}{2a}[e^{-ax^2}]_0^\infty = \frac{1}{2a} \\
\int_{-\infty}^\infty dx x e^{-ax^2} &= 0, \qquad \int_{-\infty}^\infty \text{odd} \times \text{even} = 0 \\
\int_{-\infty}^\infty dx x^2 e^{-ax^2} &= \int_{-\infty}^\infty dx x \frac{-1}{2a}(e^{-ax^2})' \\
&= \frac{-1}{2a}\left[e^{-ax^2} x\right]_{-\infty}^\infty + \frac{1}{2a}\int_{-\infty}^\infty dx e^{-ax^2} x' = \frac{1}{2a}\sqrt{\frac{\pi}{a}} \\
\int_0^\infty dx x^2 e^{-ax^2} &= \frac{1}{4a}\sqrt{\frac{\pi}{a}}
\end{aligned}
$$

$$
\int_{-\infty}^\infty dx x^3 e^{-ax^2} = 0
$$

$$
\begin{aligned}
xe^{-ax^2} &= \frac{-1}{2a}(e^{-ax^2})' \\
(fg)' &= f'g + fg', \qquad \int f'g = fg - \int fg' \\
\int_0^\infty dx x e^{-ax^2} &= \frac{-1}{2a}[e^{-ax^2}]_0^\infty = \frac{1}{2a} \\
\int_{-\infty}^\infty dx x e^{-ax^2} &= 0, \qquad \int_{-\infty}^\infty \text{odd} \times \text{even} = 0 \\
\int_{-\infty}^\infty dx x^2 e^{-ax^2} &= \int_{-\infty}^\infty dx x \frac{-1}{2a}(e^{-ax^2})' \\
&= \frac{-1}{2a}\left[e^{-ax^2} x\right]_{-\infty}^\infty + \frac{1}{2a}\int_{-\infty}^\infty dx e^{-ax^2} x' = \frac{1}{2a}\sqrt{\frac{\pi}{a}} \\
\int_0^\infty dx x^2 e^{-ax^2} &= \frac{1}{4a}\sqrt{\frac{\pi}{a}} \\
\int_{-\infty}^\infty dx x^3 e^{-ax^2} &= 0
\end{aligned}
$$

$$\int_0^\infty dx x^3 e^{-ax^2} = \int_0^\infty dx x^2 \frac{-1}{2a}(e^{-ax^2})'$$

$$= \frac{-1}{2a}\left[e^{-ax^2}x^2\right]_0^\infty + \frac{1}{2a}\int_0^\infty dx e^{-ax^2}(x^2)' = \frac{1}{a}\int_0^\infty dx x e^{-ax^2} = \frac{1}{2a^2}$$

$$\int_{-\infty}^\infty dx x^4 e^{-ax^2} = \int_{-\infty}^\infty dx x^3 \frac{-1}{2a}(e^{-ax^2})'$$

$$= \frac{-1}{2a}\left[e^{-ax^2}x^3\right]_{-\infty}^\infty + \frac{3}{2a}\int_{-\infty}^\infty dx e^{-ax^2}x^2 = \frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty dx x^4 e^{-ax^2} = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}$$

$$
\begin{aligned}
\int_0^\infty dx x^3 e^{-ax^2} &= \int_0^\infty dx x^2 \frac{-1}{2a} (e^{-ax^2})' \\
&= \frac{-1}{2a} \left[e^{-ax^2} x^2 \right]_0^\infty + \frac{1}{2a} \int_0^\infty dx e^{-ax^2} (x^2)' = \frac{1}{a} \int_0^\infty dx x e^{-ax^2} = \frac{1}{2a^2} \\
\int_{-\infty}^\infty dx x^4 e^{-ax^2} &= \int_{-\infty}^\infty dx x^3 \frac{-1}{2a} (e^{-ax^2})' \\
&= \frac{-1}{2a} \left[e^{-ax^2} x^3 \right]_{-\infty}^\infty + \frac{3}{2a} \int_{-\infty}^\infty dx e^{-ax^2} x^2 = \frac{3}{4a^2} \sqrt{\frac{\pi}{a}} \\
\int_0^\infty dx x^4 e^{-ax^2} &= \frac{3}{8a^2} \sqrt{\frac{\pi}{a}}
\end{aligned}
$$

$x^5:$

$$(e^{-ax^2})' = e^{-ax^2}(-2ax), \quad xe^{-ax^2} = -\frac{1}{2a}(e^{-ax^2})'$$

$$\int_{-\infty}^{+\infty} x^5 e^{-ax^2}\mathrm{d}x = 0, \qquad \int_{-\infty}^{+\infty}(\text{odd})(\text{even}) = 0$$

$$\int_{0}^{+\infty} x^5 e^{-ax^2}\mathrm{d}x = \int_{0}^{+\infty} x^4 \frac{-1}{2a}(e^{-ax^2})'\mathrm{d}x = -\frac{1}{2a}\int_{0}^{+\infty} x^4 (e^{-ax^2})'\mathrm{d}x$$

$$(fg)' = f'g + fg', \quad \int fg' = fg - \int f'g$$

$$-\frac{1}{2a}\int_{0}^{+\infty} x^4 (e^{-ax^2})'\mathrm{d}x = -\frac{1}{2a}\left[x^4 e^{-ax^2}\right]_0^{+\infty} + \frac{1}{2a}\int_{0}^{+\infty} 4x^3 e^{-ax^2}\mathrm{d}x$$

$$= 0 - 0 + \frac{2}{a}\int_{0}^{+\infty} x^3 e^{-ax^2}\mathrm{d}x = \frac{2}{a}\frac{1}{2a^2} = \frac{1}{a^3}$$

$x^6:$

$$(e^{-ax^2})' = e^{-ax^2}(-2ax), \quad xe^{-ax^2} = -\frac{1}{2a}(e^{-ax^2})'$$

$$\int_{-\infty}^{+\infty} x^6 e^{-ax^2}\mathrm{d}x = -\frac{1}{2a}\int_{-\infty}^{+\infty} x^5 (e^{-ax^2})'\mathrm{d}x$$

$$(fg)' = f'g + fg', \quad \int fg' = fg - \int f'g$$

$$= -\frac{1}{2a}\left[x^5 e^{-ax^2}\right]_{-\infty}^{+\infty} + \frac{1}{2a}\int_{-\infty}^{+\infty} 5x^4 e^{-ax^2}\mathrm{d}x$$

$$= 0 - 0 + \frac{5}{2a}\int_{-\infty}^{+\infty} x^4 e^{-ax^2}\mathrm{d}x = \frac{5}{2a}\frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

$$= \frac{15}{8a^3}\sqrt{\frac{\pi}{a}}$$

$$\int_{0}^{+\infty} x^6 e^{-ax^2}\mathrm{d}x = \frac{1}{2}\int_{-\infty}^{+\infty} x^6 e^{-ax^2}\mathrm{d}x = \frac{15}{16a^3}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{+\infty} \sqrt{x} e^{-ax} \mathrm{d}x :$$

$$x = X^2, \quad \frac{\mathrm{d}x}{\mathrm{d}X} = 2X, \quad \mathrm{d}x = 2X\mathrm{d}X$$

$$\int_0^{+\infty} \sqrt{x} e^{-ax} \mathrm{d}x = \int_0^{+\infty} X e^{-aX^2} 2X \mathrm{d}X$$

$$= 2\int_0^{+\infty} X^2 e^{-aX^2} \mathrm{d}X = 2\frac{1}{4a}\sqrt{\frac{\pi}{a}} = \frac{1}{2a}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{+\infty} x^{3/2} e^{-ax} \mathrm{d}x = \int_0^{+\infty} X^3 e^{-aX^2} 2X \mathrm{d}X = 2\int_0^{+\infty} X^4 e^{-aX^2} \mathrm{d}X$$

$$= 2\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{+\infty} x^{5/2} e^{-ax} \mathrm{d}x = \int_0^{+\infty} X^5 e^{-aX^2} 2X \mathrm{d}X = 2\int_0^{+\infty} X^6 e^{-aX^2} \mathrm{d}X$$

$$= 2\frac{15}{16a^3}\sqrt{\frac{\pi}{a}} = \frac{15}{8a^3}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{+\infty} F(\epsilon)\mathrm{d}\epsilon = 1, \quad F(\epsilon) = \frac{2\pi}{(\pi k_\mathrm{B}T)^{3/2}}\epsilon^{1/2}e^{-\epsilon/(k_\mathrm{B}T)}$$

$$\langle\epsilon\rangle = \int_0^{+\infty} \epsilon F(\epsilon)\mathrm{d}\epsilon = \frac{3}{2}k_\mathrm{B}T$$

$$\langle\epsilon^2\rangle = \int_0^{+\infty} \epsilon^2 F(\epsilon)\mathrm{d}\epsilon = \frac{15}{4}(k_\mathrm{B}T)^2$$

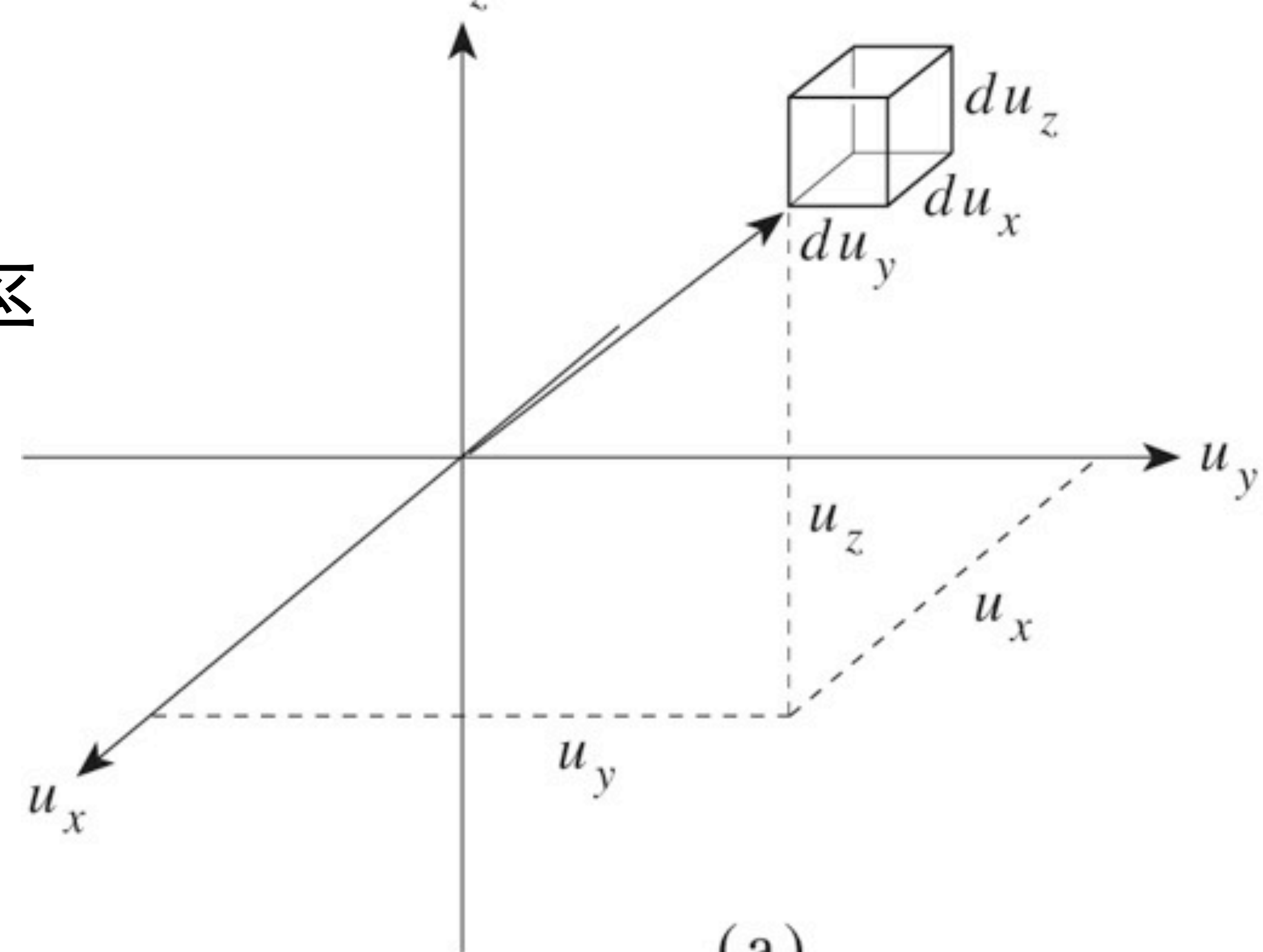
$$\sigma_\epsilon^2 = \langle\epsilon^2\rangle - \langle\epsilon\rangle^2 = \frac{15-9}{4}(k_\mathrm{B}T)^2$$

$$\sigma_\epsilon = \sqrt{\frac{3}{2}}k_\mathrm{B}T$$

$$\frac{\sigma_\epsilon}{\langle\epsilon\rangle} = \frac{\sqrt{\frac{3}{2}}k_\mathrm{B}T}{\frac{3}{2}k_\mathrm{B}T} = \sqrt{\frac{2}{3}} = 0.8165...$$

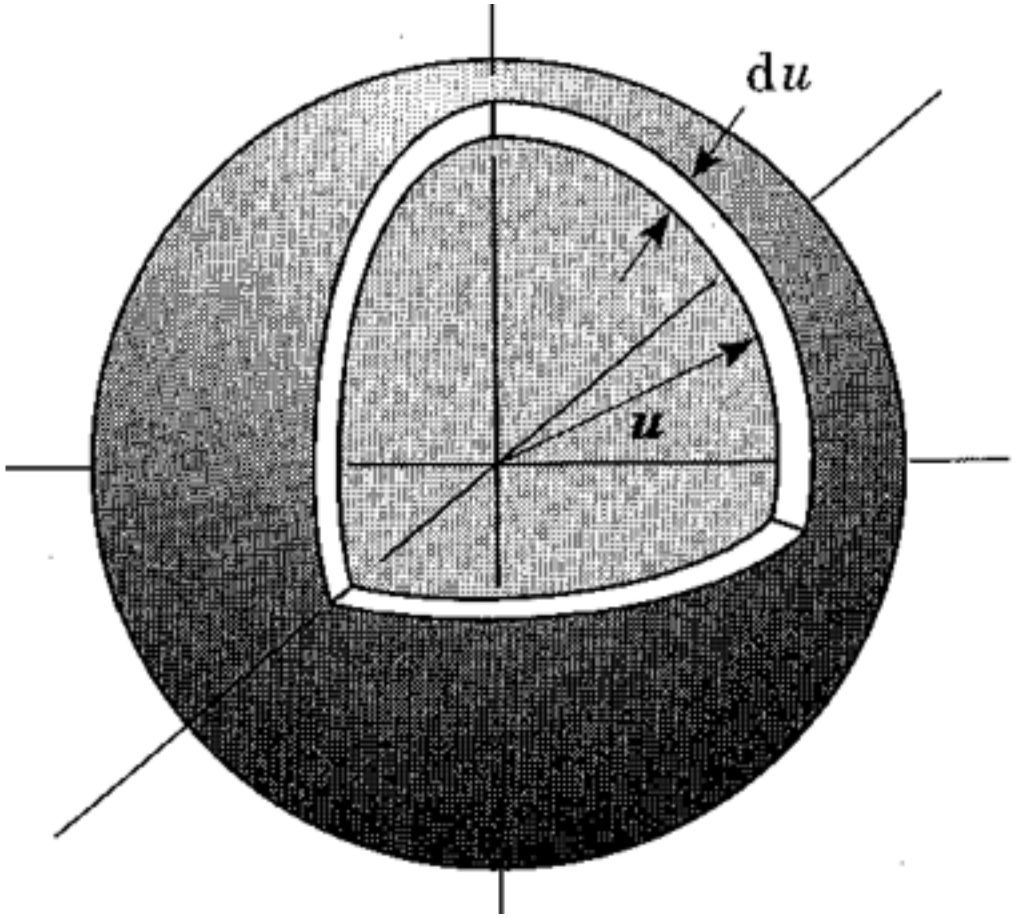
揺らぎは小さくない

$$u_x \sim u_x + du_x$$
$$u_y \sim u_y + du_y$$ にある確率
$$u_z \sim u_z + du_z$$



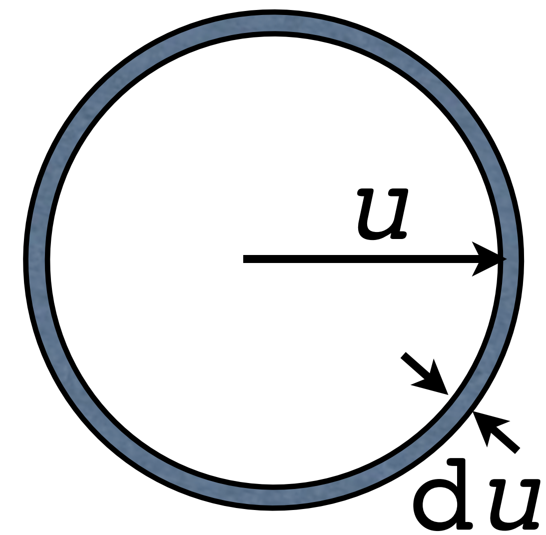
(a)

$$\begin{aligned} f(u_x)f(u_y)f(u_z)du_x du_y du_z &= \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2} \exp\left[-\frac{m(u_x^2+u_y^2+u_z^2)}{2k_{\mathrm{B}}T}\right] du_x du_y du_z \\ &= \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2} \exp\left(-\frac{mu^2}{2k_{\mathrm{B}}T}\right) du_x du_y du_z \end{aligned}$$



u~u+du
にある領域の体積dV

$dV = 4\pi u^2 du$



球の表面積 $4\pi u^2$

$$
\begin{aligned}
(V + dV) - V &= \frac{4\pi}{3}[(u + du)^3 - u^3] \\
&= \frac{4\pi}{3}[u^3 + 3u^2du + 3u(du)^2 + (du)^3 - u^3] \\
&\simeq \frac{4\pi}{3}3u^2du = 4\pi u^2 du
\end{aligned}
$$

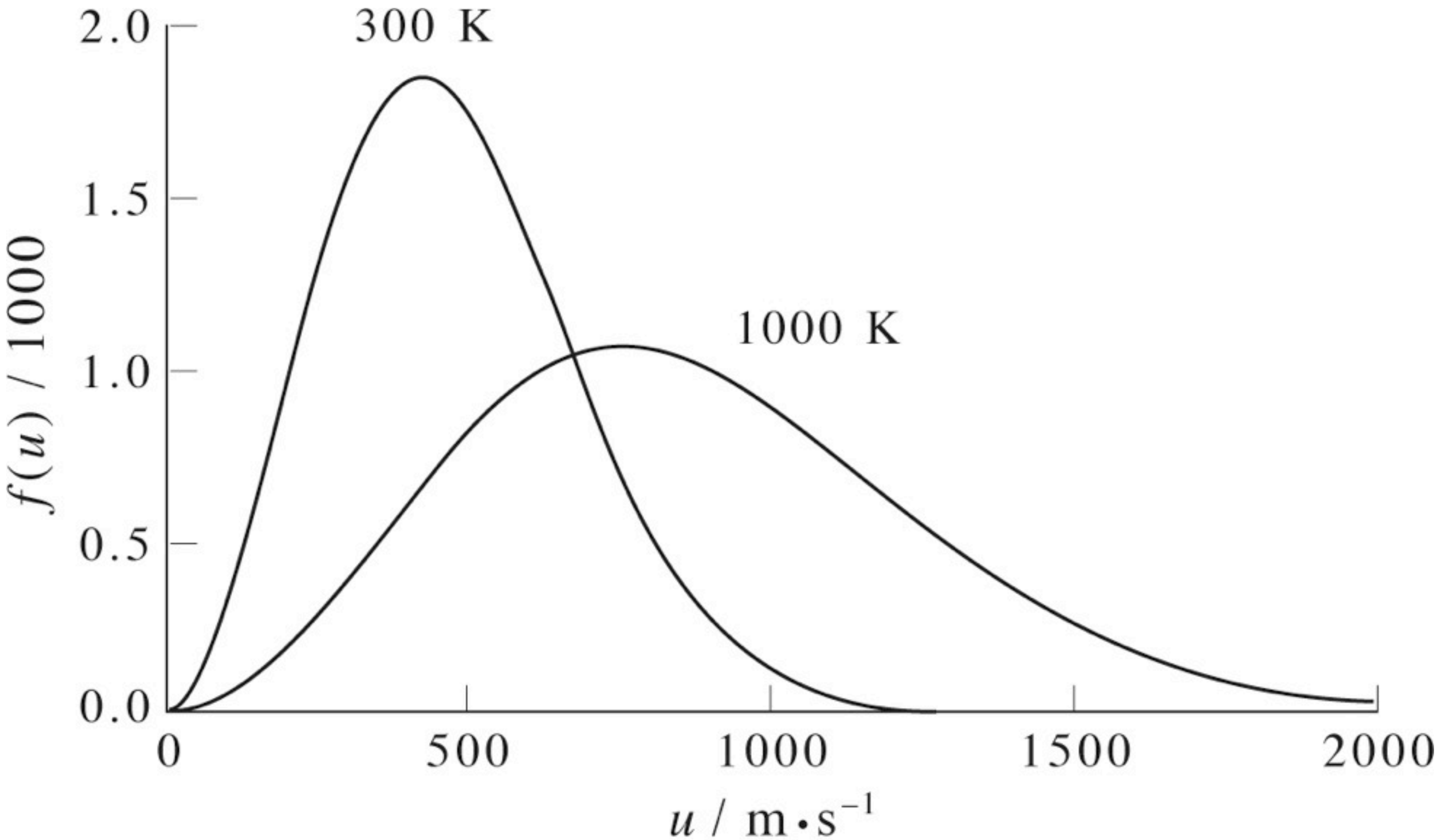
$$
\begin{aligned}
f(u_x)f(u_y)f(u_z)du_x du_y du_z &= \left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2} \exp\left[-\frac{m(u_x^2+u_y^2+u_z^2)}{2k_\mathrm{B}T}\right] du_x du_y du_z \\
&= \left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2} \exp\left(-\frac{mu^2}{2k_\mathrm{B}T}\right) du_x du_y du_z \\
&\Rightarrow
\end{aligned}
$$

$$
F(u)du = 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2} u^2 \exp\left(-\frac{mu^2}{2k_\mathrm{B}T}\right) du
$$

Mawell-Boltzmann分布

$$
\begin{aligned}
f(u_x)f(u_y)f(u_z)du_x du_y du_z &= \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2} \exp\left[-\frac{m(u_x^2+u_y^2+u_z^2)}{2k_{\mathrm{B}}T}\right] du_x du_y du_z \\
&= \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2} \exp\left(-\frac{mu^2}{2k_{\mathrm{B}}T}\right) du_x du_y du_z \\
&\Rightarrow \\
F(u)du &= 4\pi \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2} u^2 \exp\left(-\frac{mu^2}{2k_{\mathrm{B}}T}\right) du
\end{aligned}
$$

Mawell-Boltzmann分布



300 K
1000 K
2.0
1.5
1.0
0.5
0.0
f(u) / 1000
0
500
1000
1500
2000
u / m•s⁻¹

規格化 $$\int_0^\infty F(u)du = 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\int_0^\infty u^2 e^{-mu^2/(2k_\mathrm{B}T)}du$$

$$= 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\frac{2k_\mathrm{B}T}{4m}\sqrt{\frac{2\pi k_\mathrm{B}T}{m}}$$

$$= \pi\frac{m}{2\pi k_\mathrm{B}T}\frac{2k_\mathrm{B}T}{m} = 1$$

平均速度

$$\int_0^\infty uF(u)du = 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\int_0^\infty u^3 e^{-mu^2/(2k_\mathrm{B}T)}du$$

$$= 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\frac{(2k_\mathrm{B}T)^2}{2m^2}$$

$$= 2\sqrt{\frac{m}{2\pi k_\mathrm{B}T}}\frac{4k_\mathrm{B}T}{2m} = \sqrt{\frac{8k_\mathrm{B}T}{\pi m}}$$

298 K N_2 475 m s^{-1}

規格化 $$\int_0^\infty F(u)du = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\int_0^\infty u^2 e^{-mu^2/(2k_B T)}du$$
$$= 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\frac{2k_B T}{4m}\sqrt{\frac{2\pi k_B T}{m}}$$
$$= \pi\frac{m}{2\pi k_B T}\frac{2k_B T}{m} = 1$$

平均速度

$$\int_0^\infty uF(u)du = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\int_0^\infty u^3 e^{-mu^2/(2k_B T)}du$$
$$= 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}\frac{(2k_B T)^2}{2m^2}$$
$$= 2\sqrt{\frac{m}{2\pi k_B T}}\frac{4k_B T}{2m} = \sqrt{\frac{8k_B T}{\pi m}}$$

298 K N_2 475 m s^{-1}

$$
\begin{aligned}
\int_0^\infty u^2 F(u)du &= 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\int_0^\infty u^4 e^{-mu^2/(2k_\mathrm{B}T)}du \\
&= 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)^{3/2}\frac{3(2k_\mathrm{B}T)^2}{8m^2}\sqrt{\frac{\pi(2k_\mathrm{B}T)}{m}} \\
&= 4\pi\left(\frac{m}{2\pi k_\mathrm{B}T}\right)\frac{3(2k_\mathrm{B}T)^2}{8m^2} = 3\frac{k_\mathrm{B}T}{m}
\end{aligned}
$$

$$(1/2)m\langle u^2\rangle = (3/2)k_\mathrm{B}T$$

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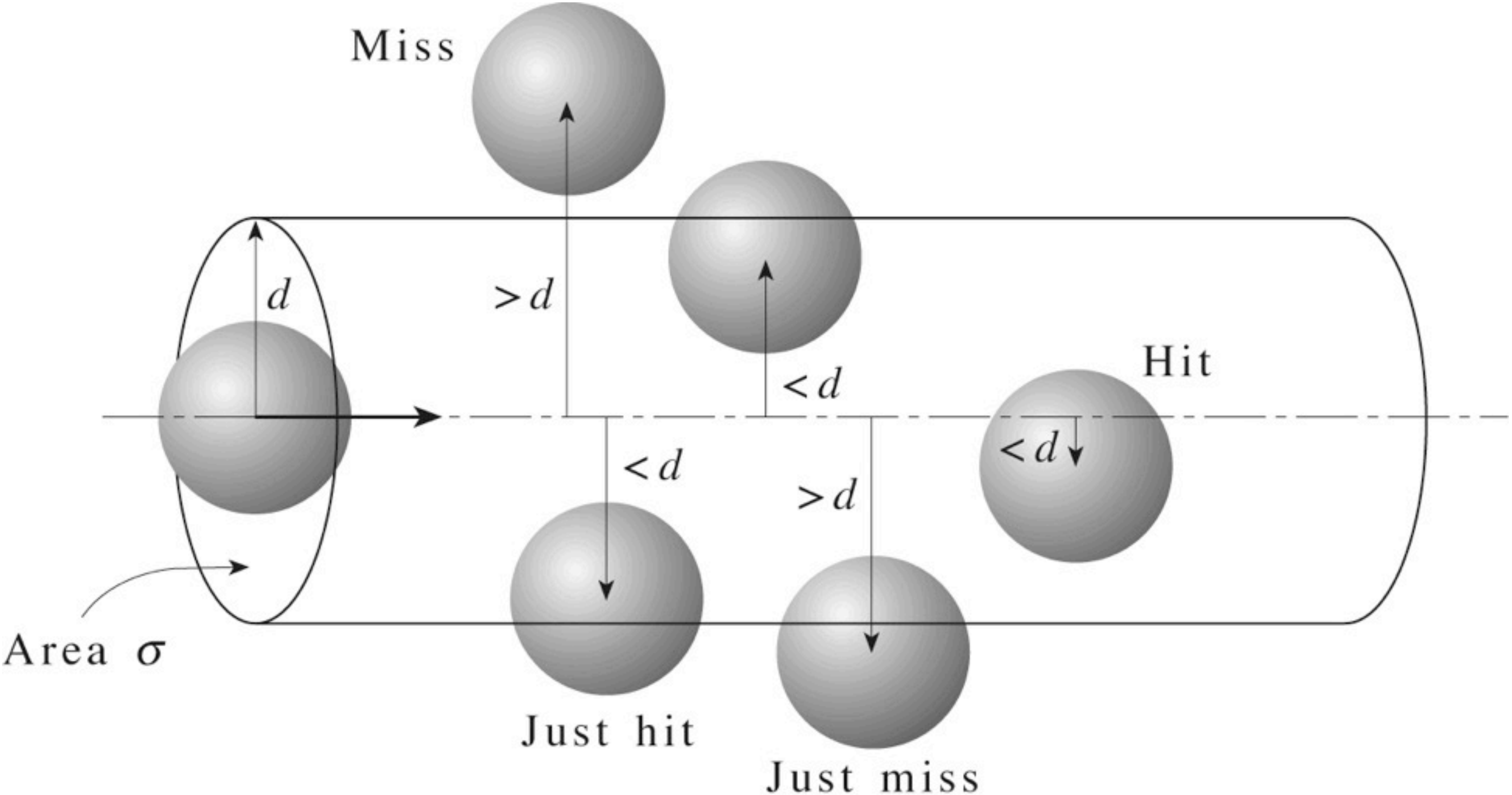
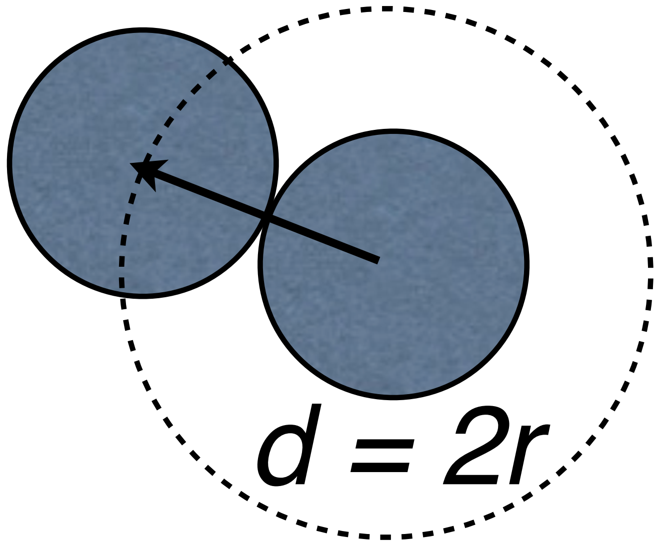
$$(1/2)m\langle u^2\rangle = (3/2)k_{\mathrm{B}}T$$

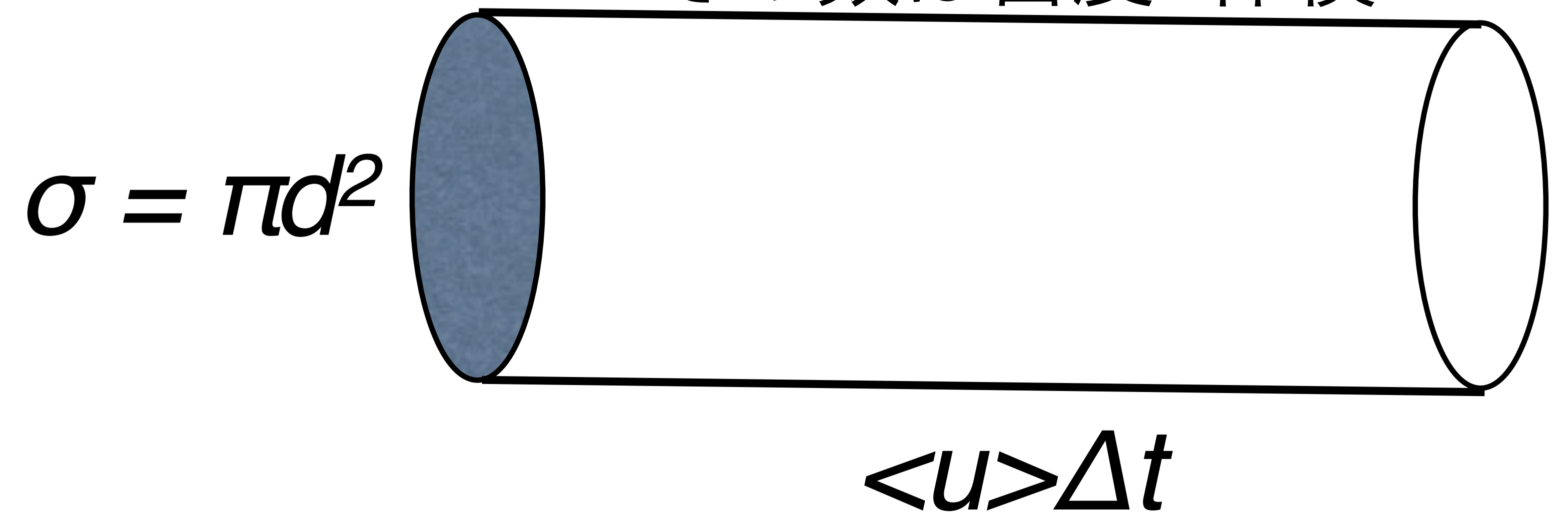
衝突頻度 z 平均自由行程 l: $l = \langle u \rangle / z$





円筒内にあるすべての分子と衝突

その数は密度×体積



$\rho\sigma<u>\Delta t$ 回 Δtの時間に衝突

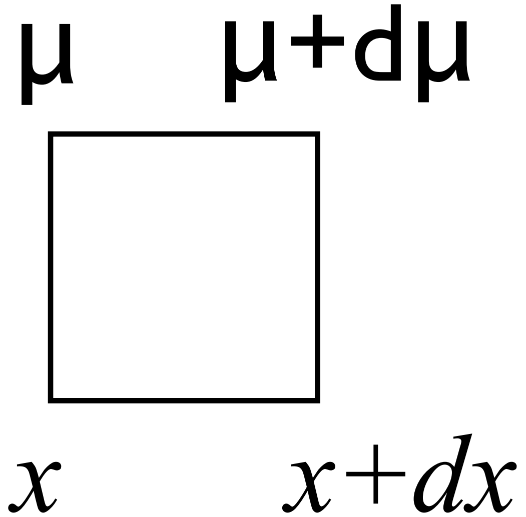
衝突頻度 z　単位時間当たり $z = \rho\sigma<u>$ 回

1 atm 298 K N_2　$z = 5\text{x}10^9\ \text{s}^{-1}$

平均自由行程 l:　　$l = <u> / z$

1 atm 298 K N_2　l=70 nm 分子直径の1000倍

拡散:化学ポテンシャルの不均一性



$$dG = -SdT + VdP + \sum_i \mu_i dN_i$$

$$\begin{aligned} dw_{\max}^{\text{non-expansion}} &= d\mu_i = \left(\frac{\partial \mu_i}{\partial x}\right)_{P,T} dx \\ &= -F_i dx \end{aligned}$$

仕事＝力×距離

$$F_i = -\left(\frac{\partial \mu_i}{\partial x}\right)_{P,T}$$

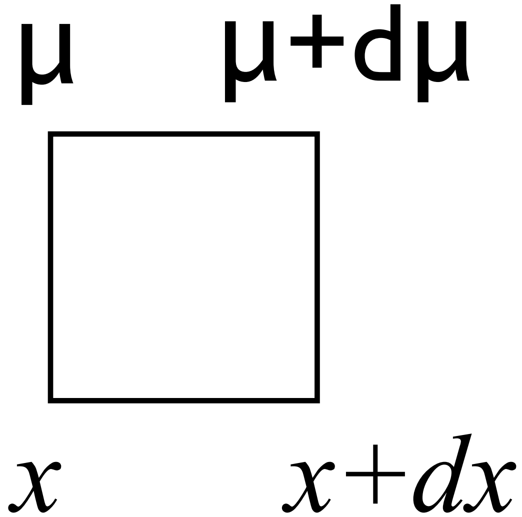
粘性流体中では速度は力に比例する

$$v_i = B_i F_i$$

流速[単位時間に単位面積を移動する物質量]

$$\begin{aligned} J_i &= c_i v_i = c_i B_i F_i \\ &= -c_i B_i \left(\frac{\partial \mu_i}{\partial x}\right)_{P,T} \\ &= -c_i B_i \left(\frac{\partial \mu_i}{\partial c_i}\right)_{P,T} \frac{\partial c_i}{\partial x} \end{aligned}$$

拡散:化学ポテンシャルの不均一性



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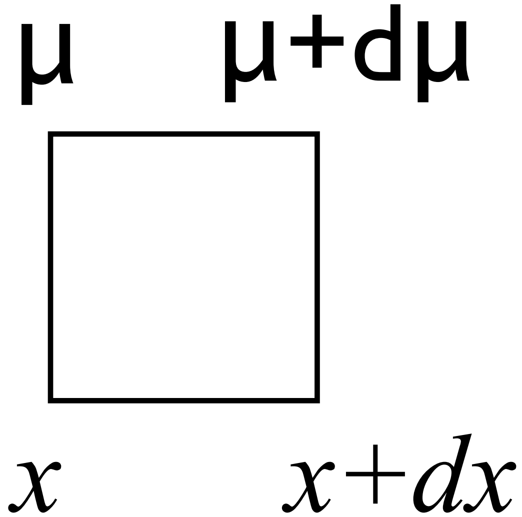

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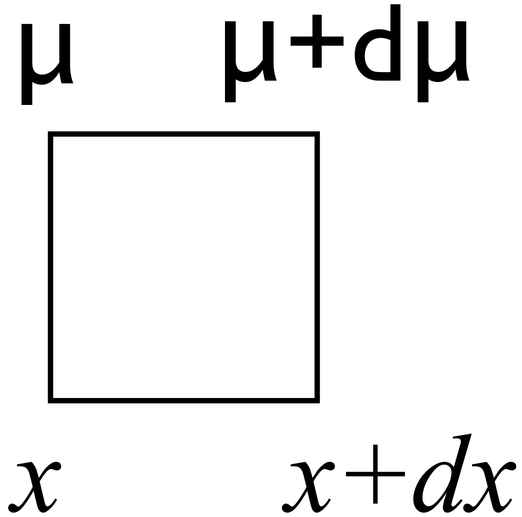

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仕事＝力×距離

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粘性流体中では速度は力に比例する $v_i = B_i F_i$

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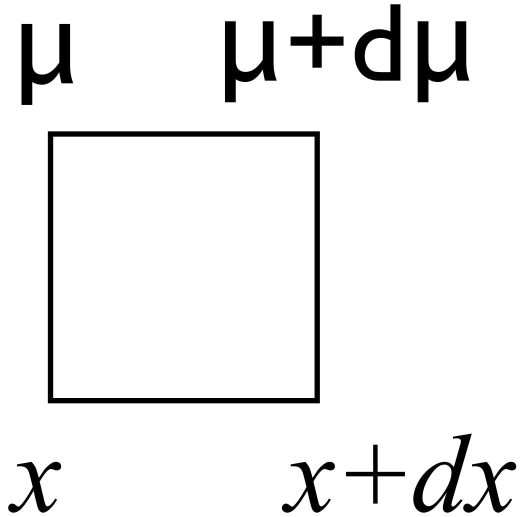

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$$J_i = -D_i \frac{\partial c_i}{\partial x}$$

Fickの第一法則

$$D_i = c_i B_i \left(\frac{\partial \mu_i}{\partial c_i}\right)_{P,T}$$

$$\mu_i = \mu^0 + kT \ln \gamma_i c_i$$

$$D_i = kTB_i\left(1 + \frac{c_i}{\gamma_i}\frac{\partial \gamma_i}{\partial c_i}\right)$$

F_i : force, v_i : velocity, J_i : flux

B_i : mobility, D_i : diffusion constant

γ_i : activity coefficient

$$
\begin{aligned}
J_i &= c_i v_i = c_i B_i F_i \\
&= -c_i B_i \left(\frac{\partial \mu_i}{\partial x}\right)_{P,T} \\
&= -c_i B_i \left(\frac{\partial \mu_i}{\partial c_i}\right)_{P,T} \frac{\partial c_i}{\partial x} \\
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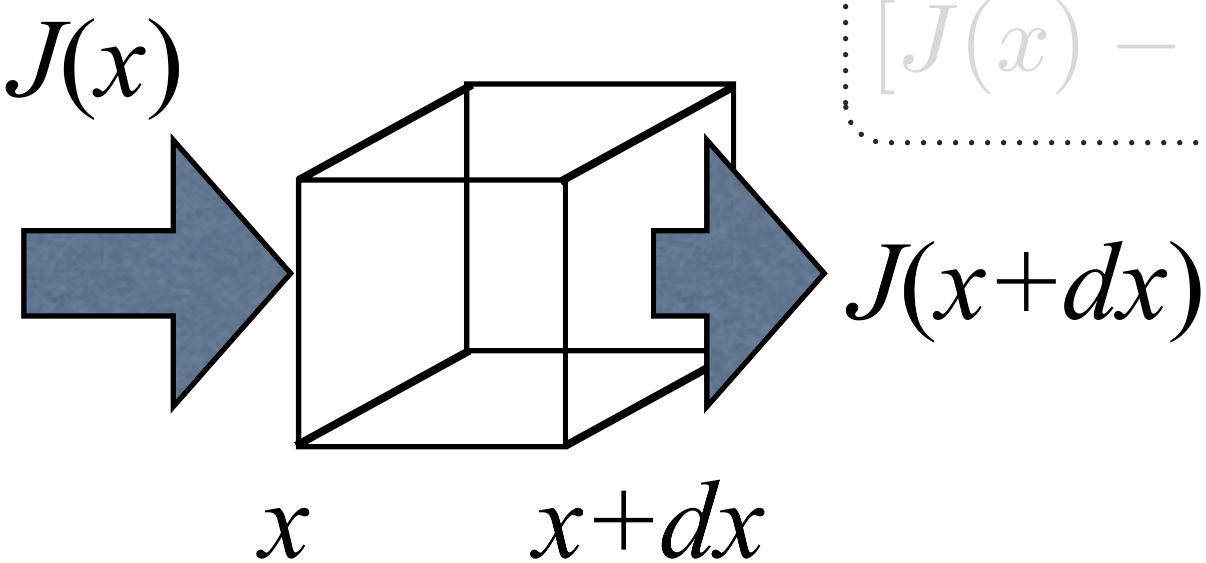


$$[J(x) - J(x+dx)]S = \frac{\partial c(x,t)}{\partial t} dV$$

$$S dx = dV$$

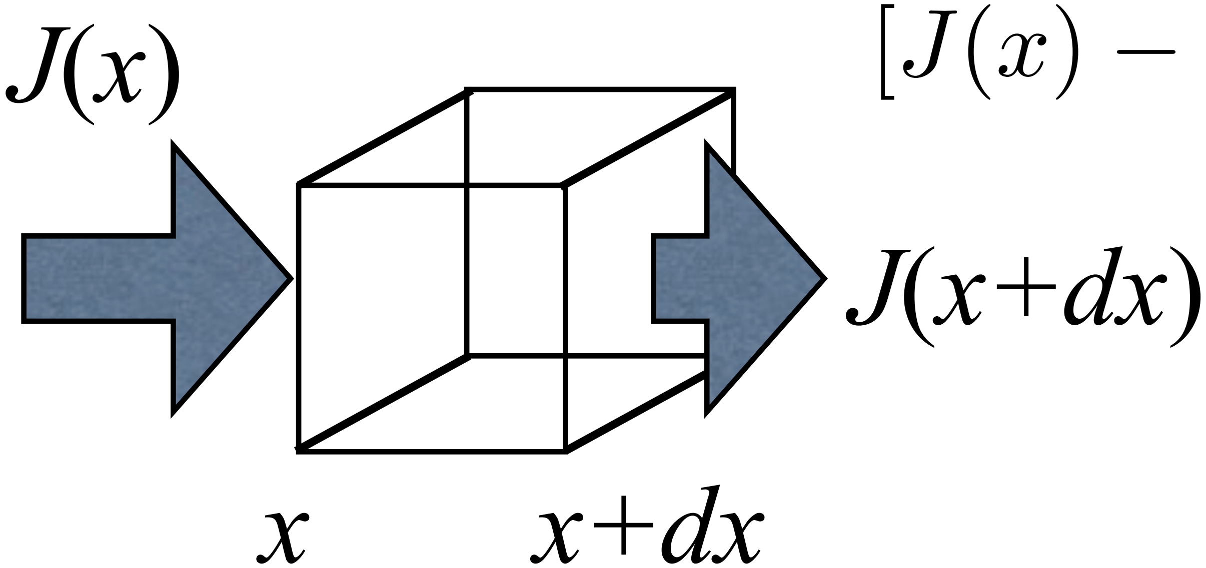
$$J(x) = -D \frac{\partial c(x,t)}{\partial x}$$

$$J(x+dx) \simeq J(x) + \frac{dJ(x)}{dx} dx$$

$$D \frac{\partial^2 c(x,t)}{\partial x^2} dx S = \frac{\partial c(x,t)}{\partial t} dV$$

$$\frac{\partial c(x,t)}{\partial t} = D \frac{\partial^2 c(x,t)}{\partial x^2}$$

拡散方程式：Fickの第二法則



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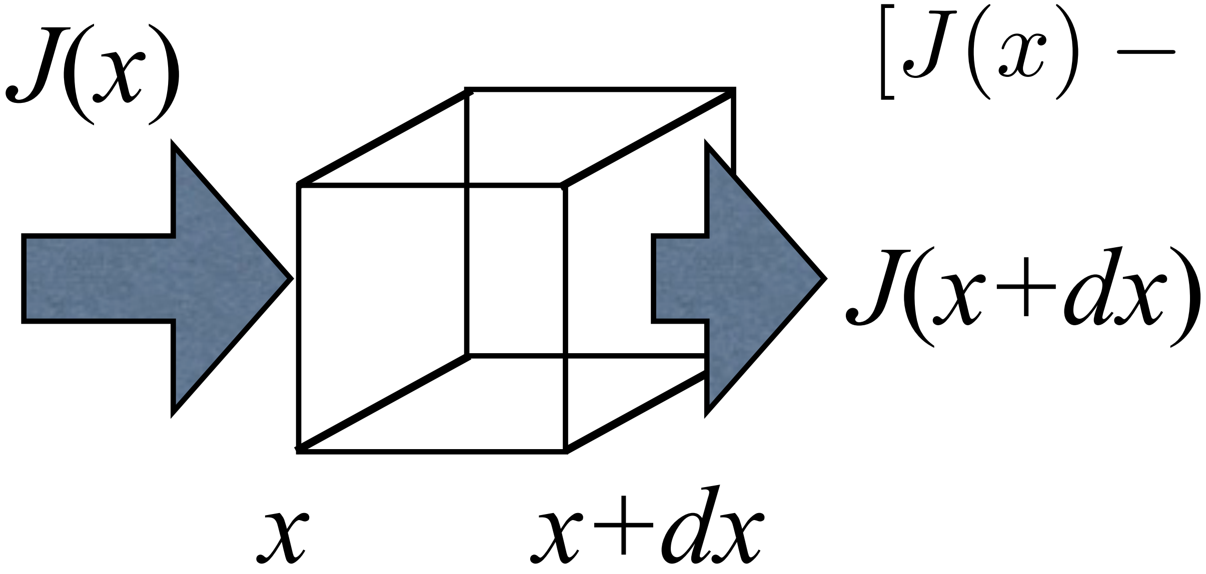

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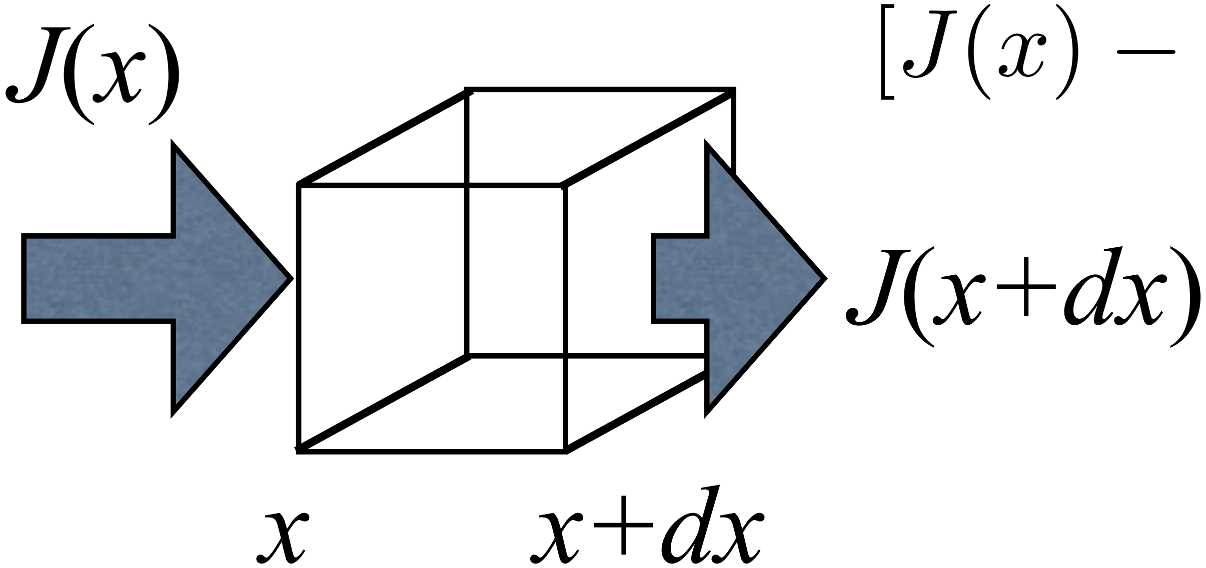

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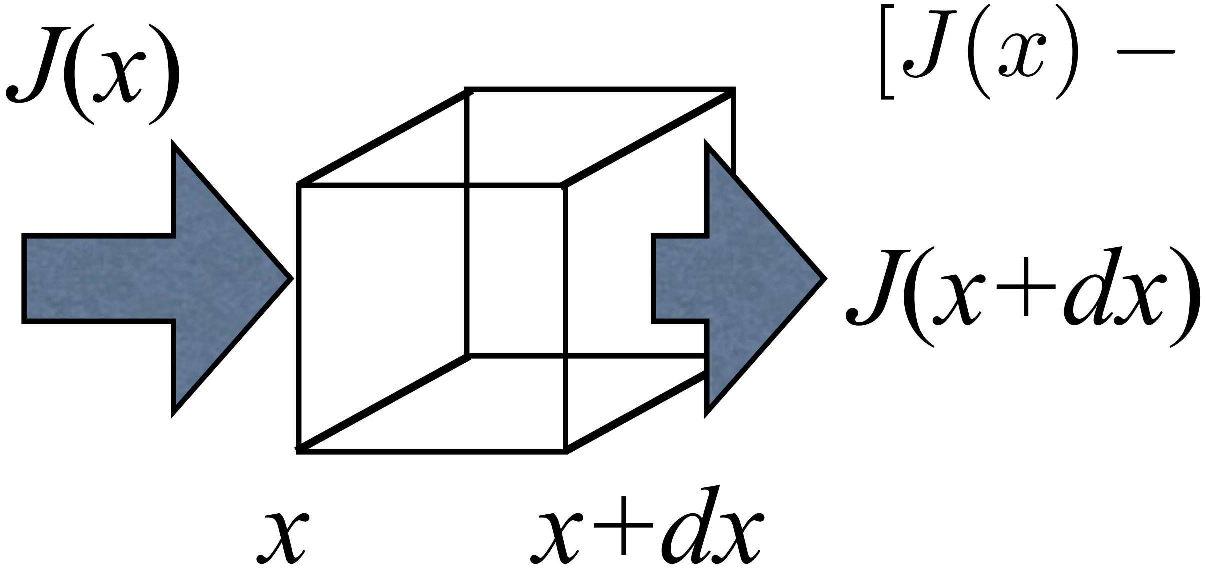

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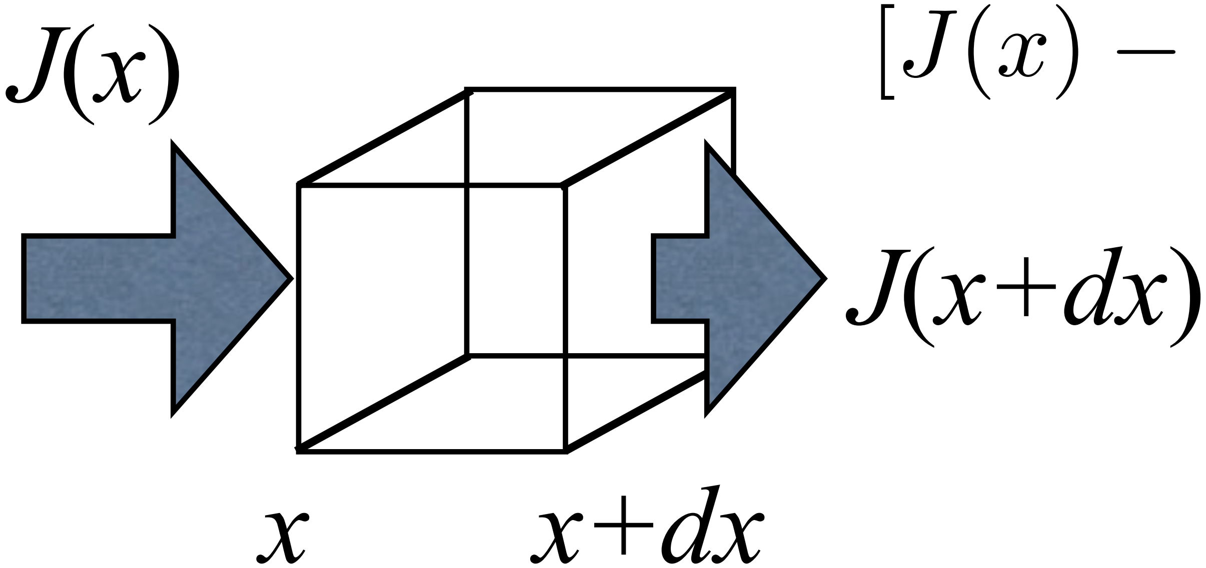

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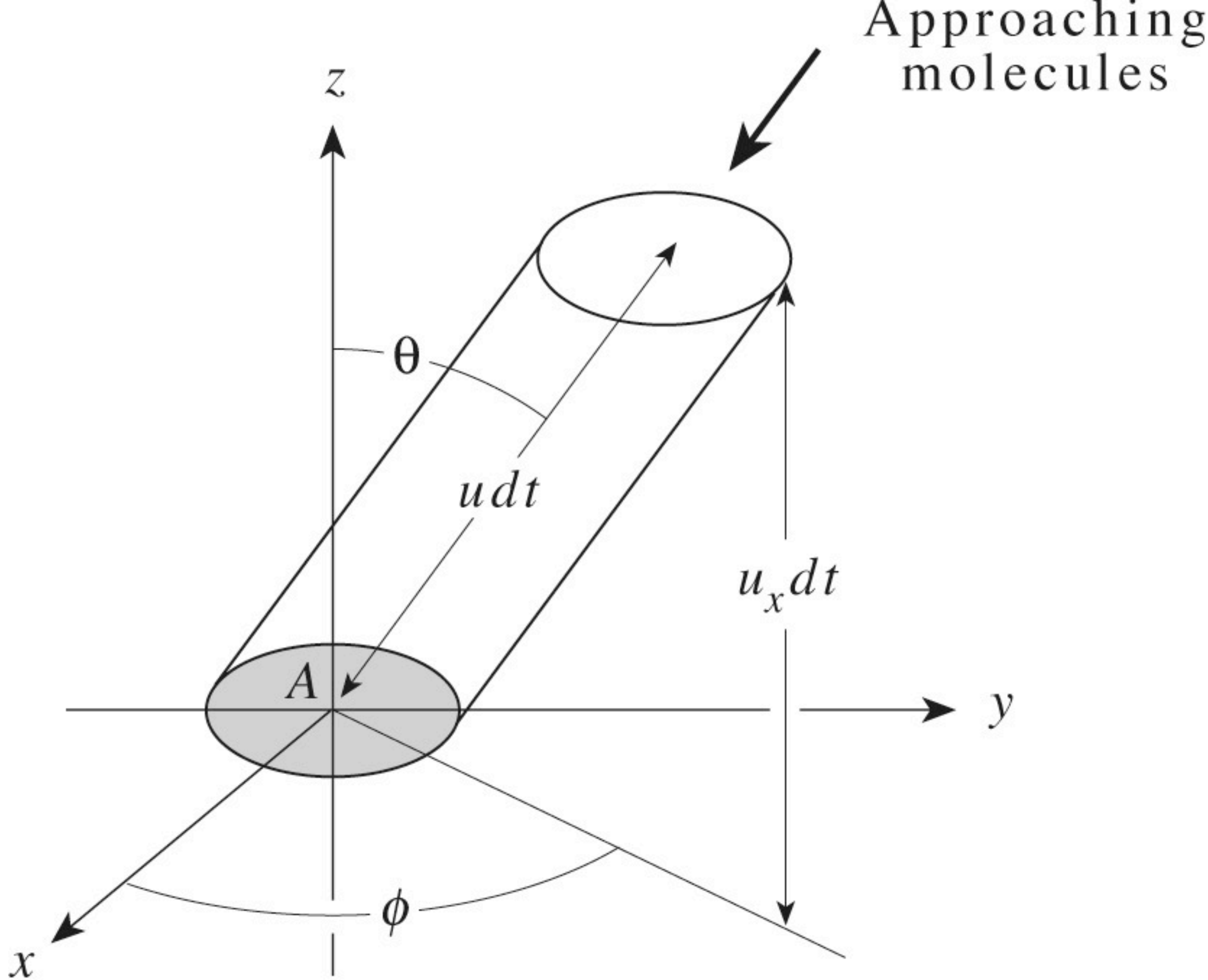
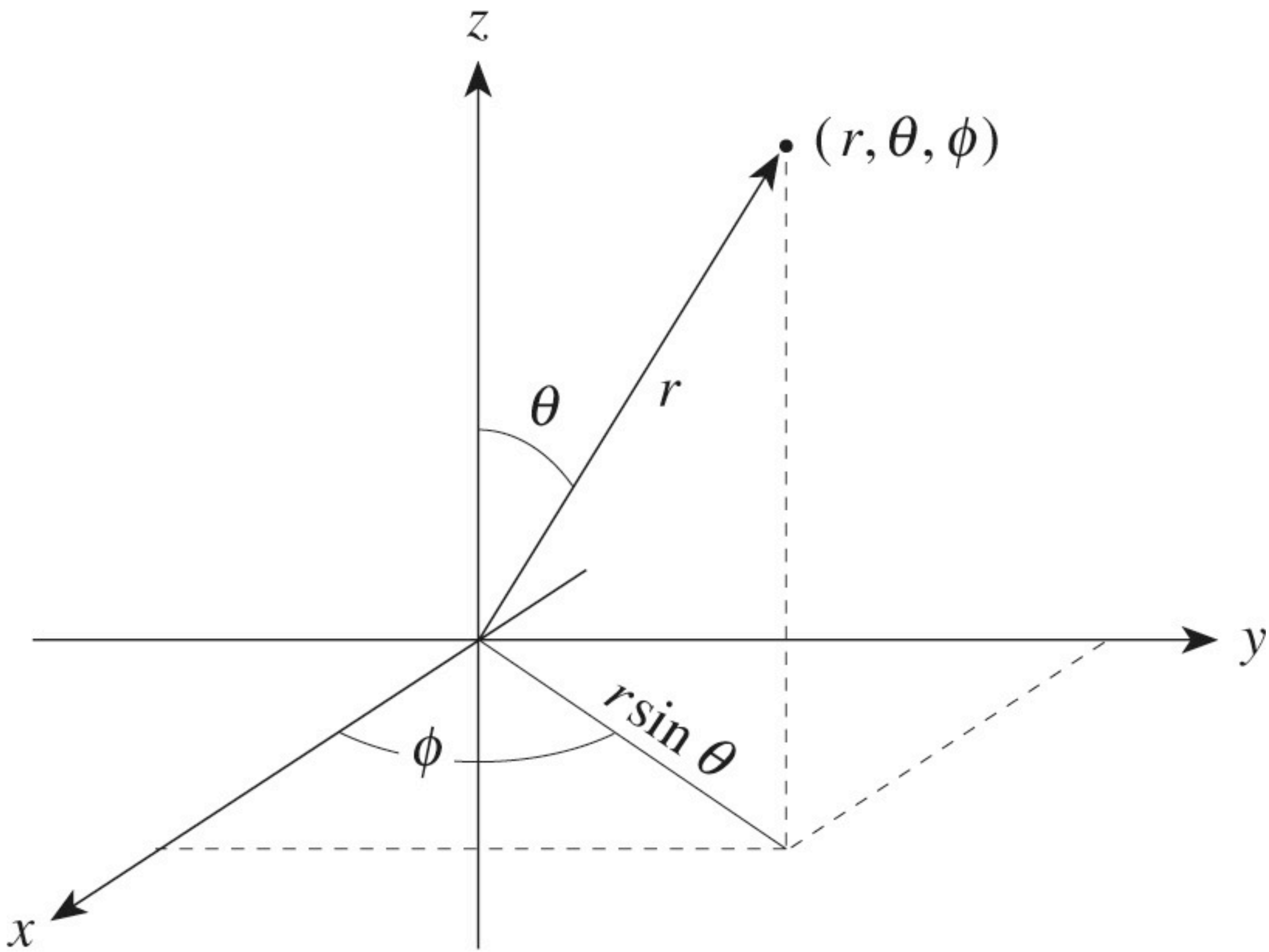
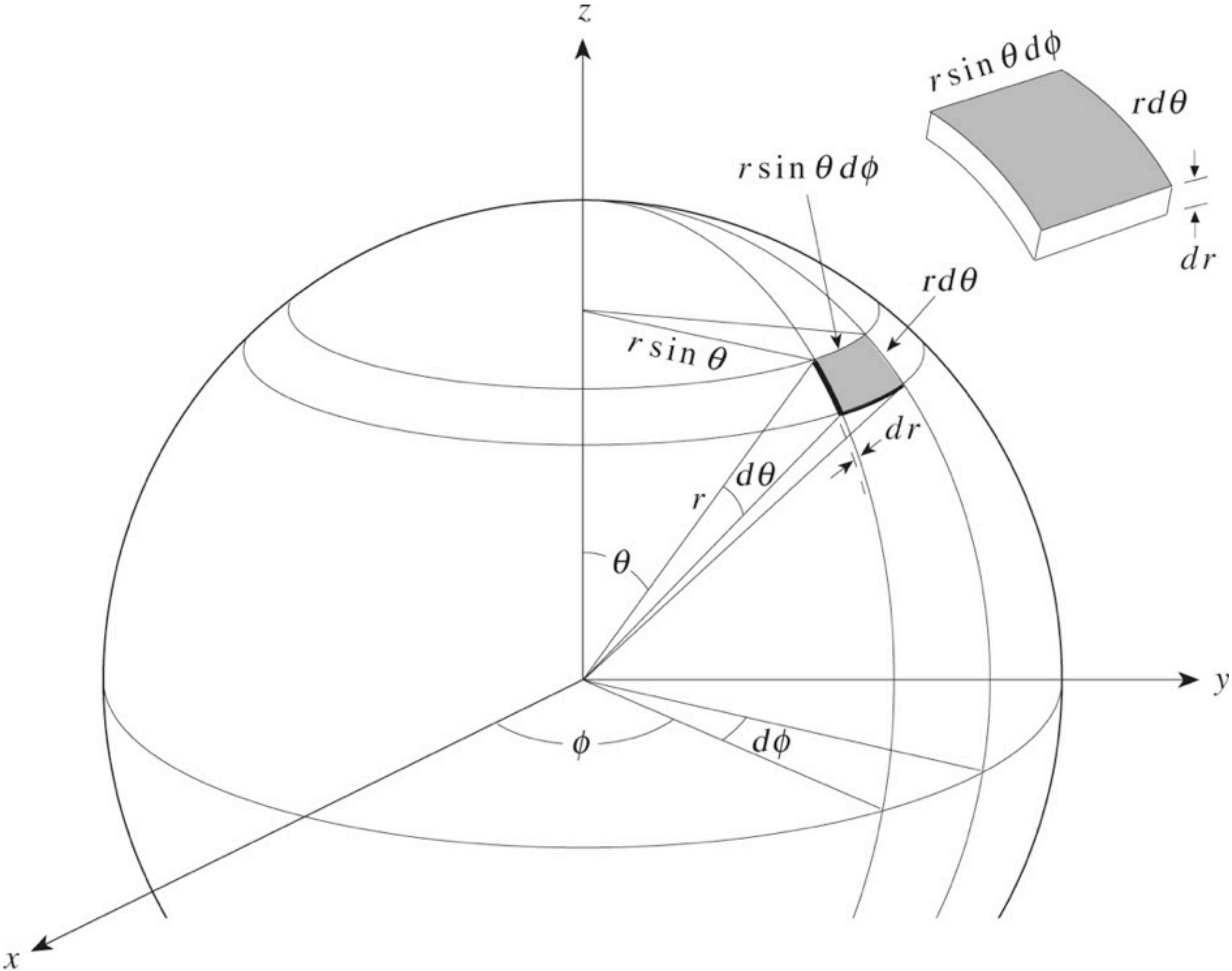
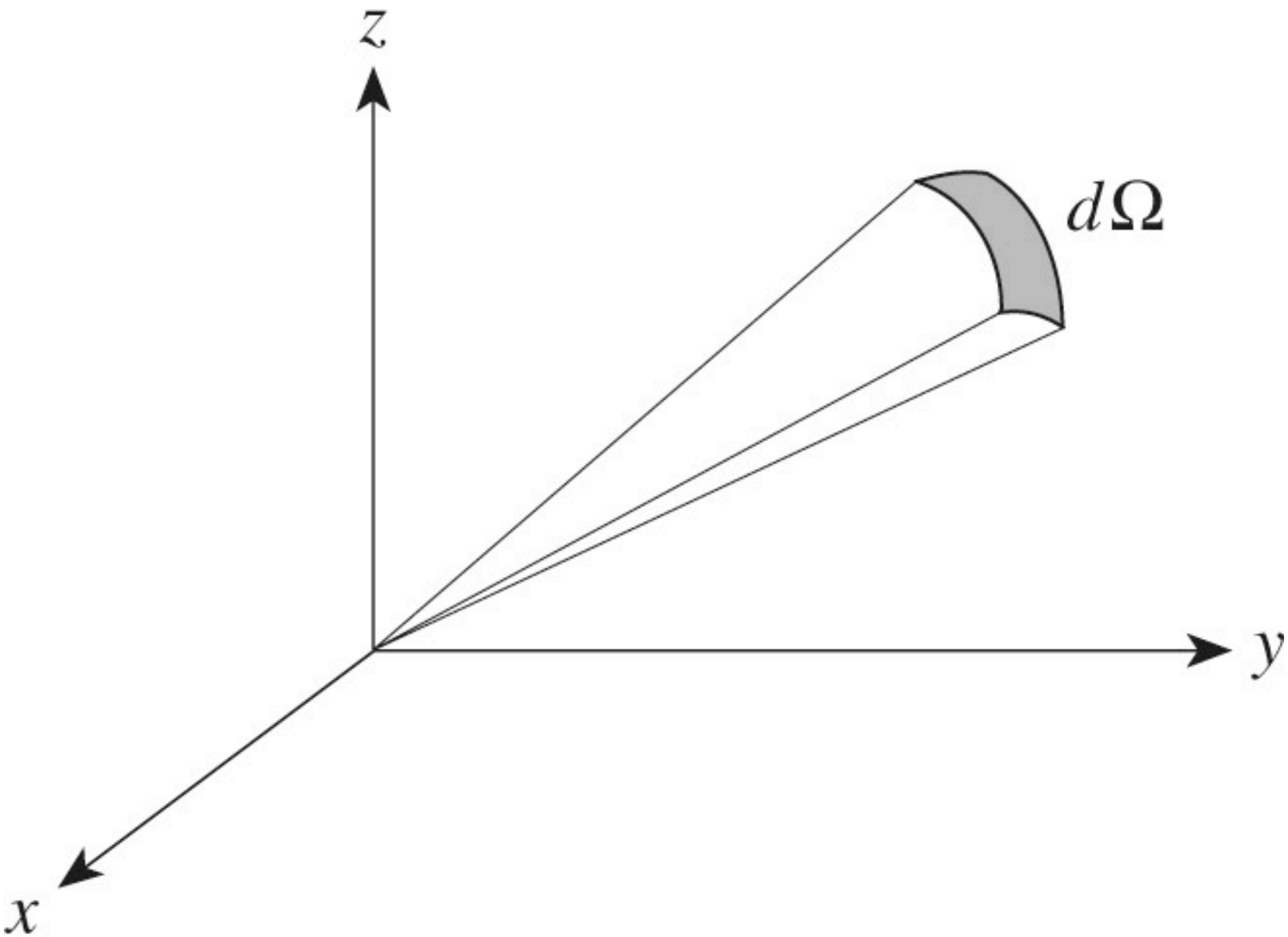

Approaching
molecules
z
θ
udt
$u_x dt$
A
y
x
φ


z
(r, θ, φ)
θ
r
y
r sin θ
φ
x



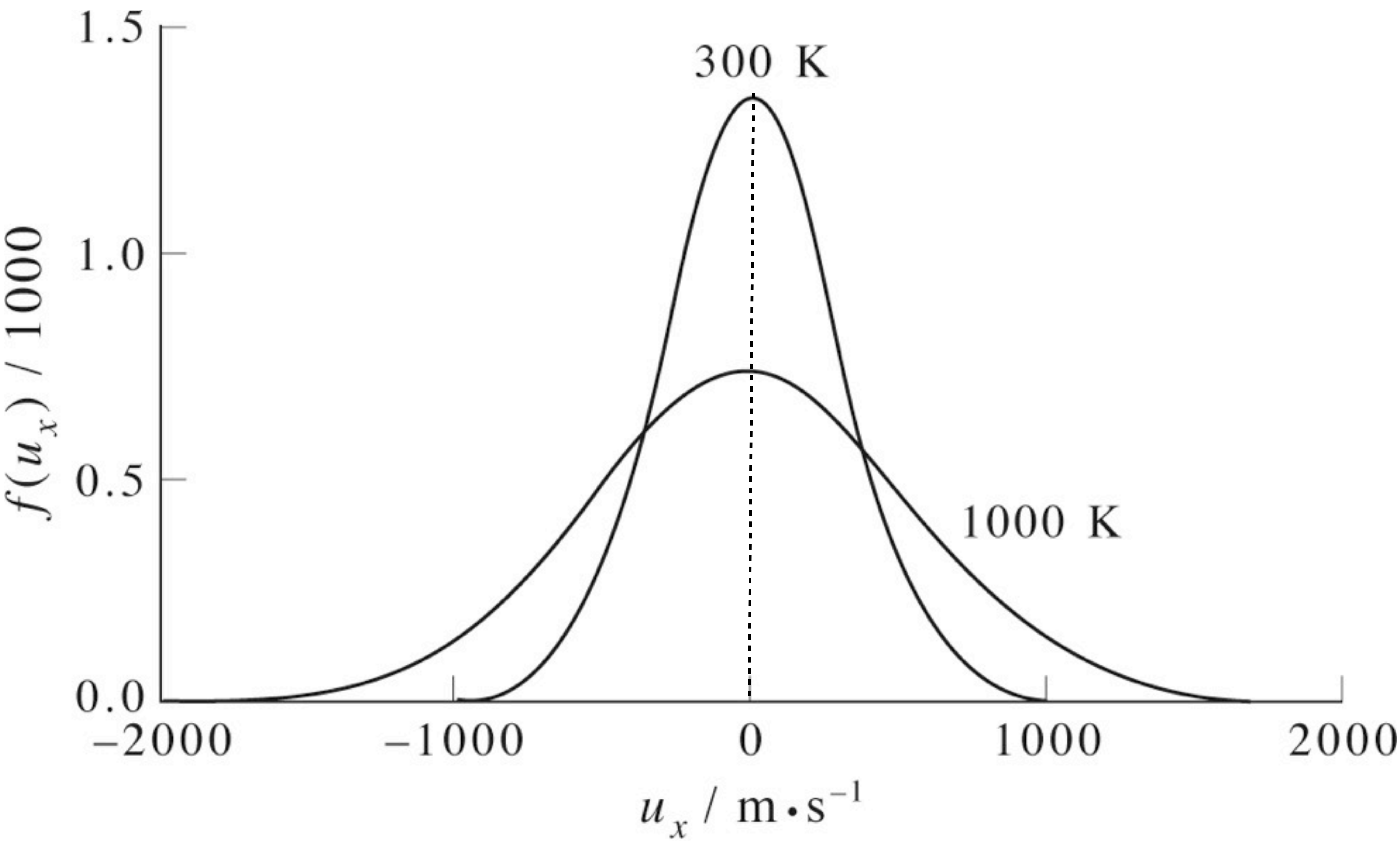
z
r sin θ dφ
rdθ
dr
r sin θ dφ
rdθ
r sin θ
dr
dθ
r
θ
y
φ
dφ
x



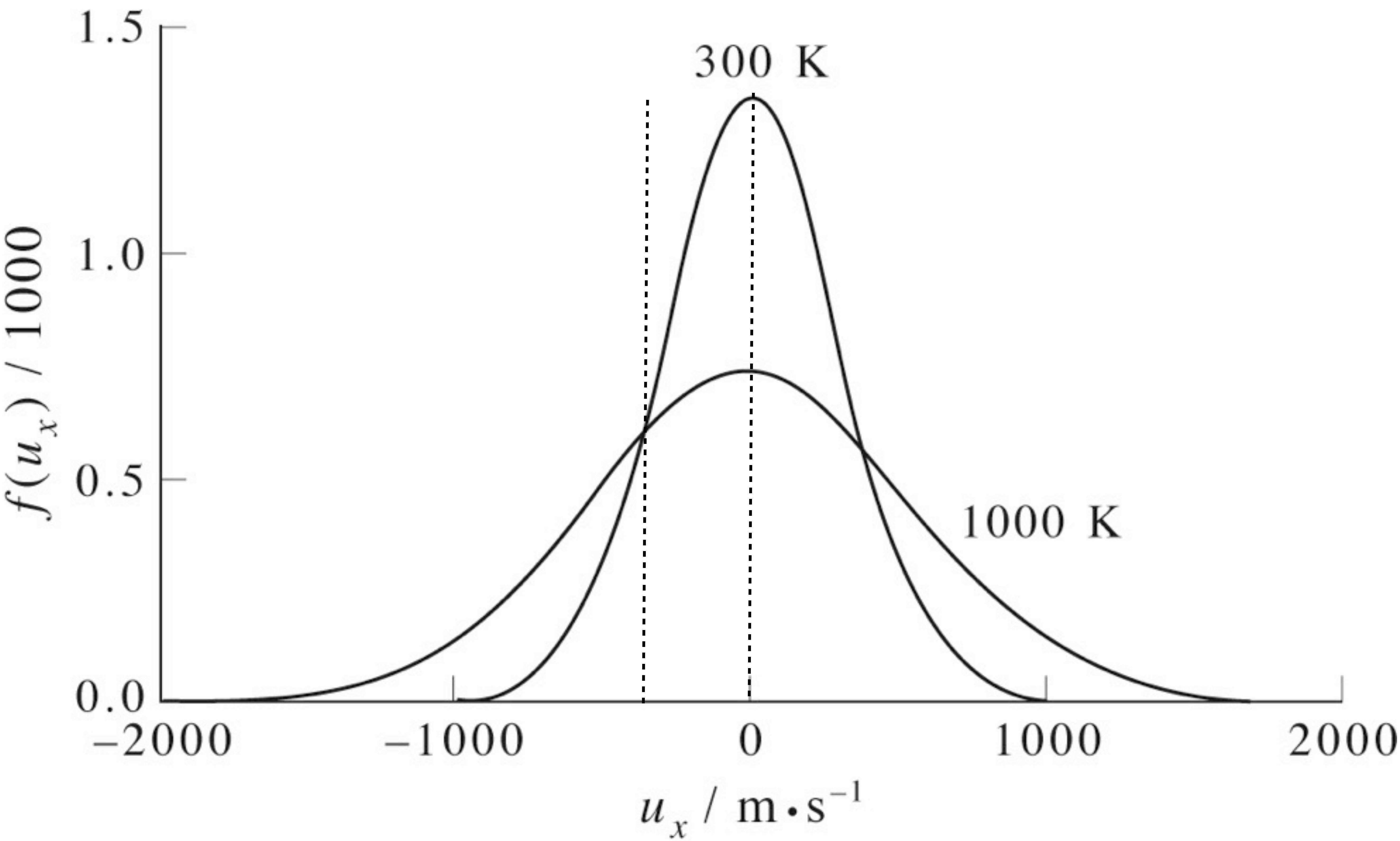
面積：r^2dΩ

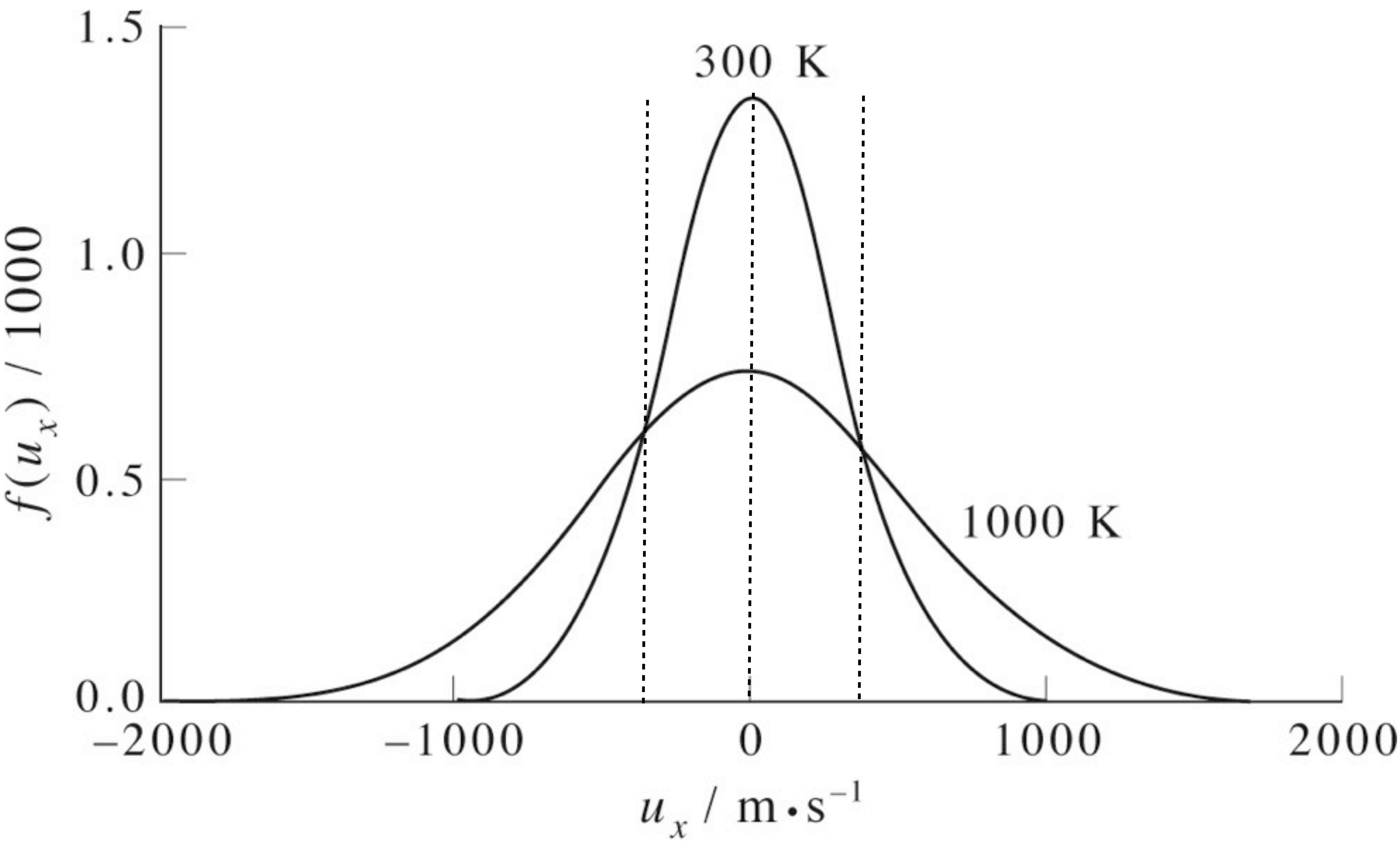
$$r^2 d\Omega = r^2 \sin\theta d\theta d\phi, \quad d\Omega = \sin\theta d\theta d\phi$$

$$\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 2\pi[-\cos\theta]_0^{\pi} = 2\pi[-(-1)+1] = 4\pi$$

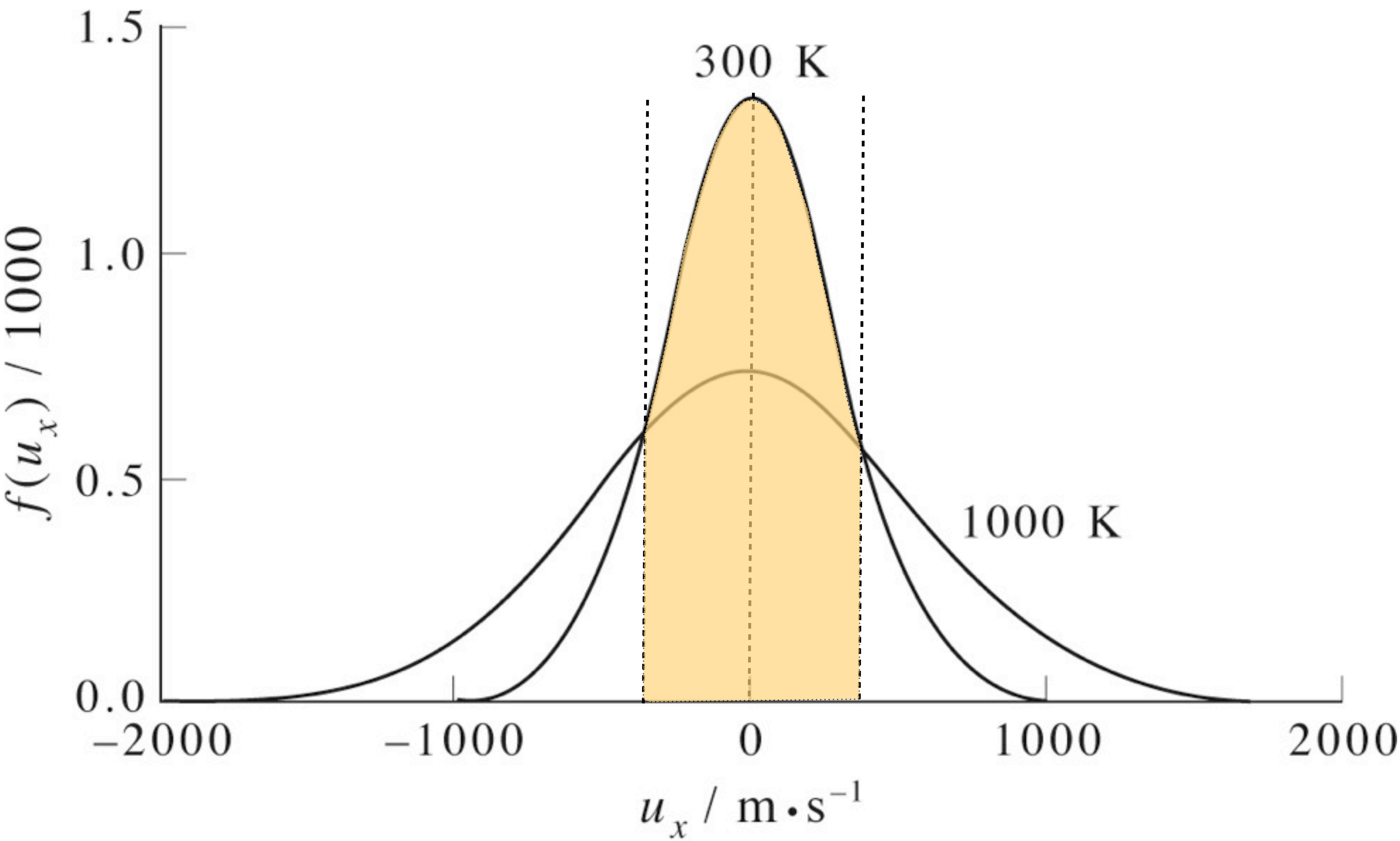


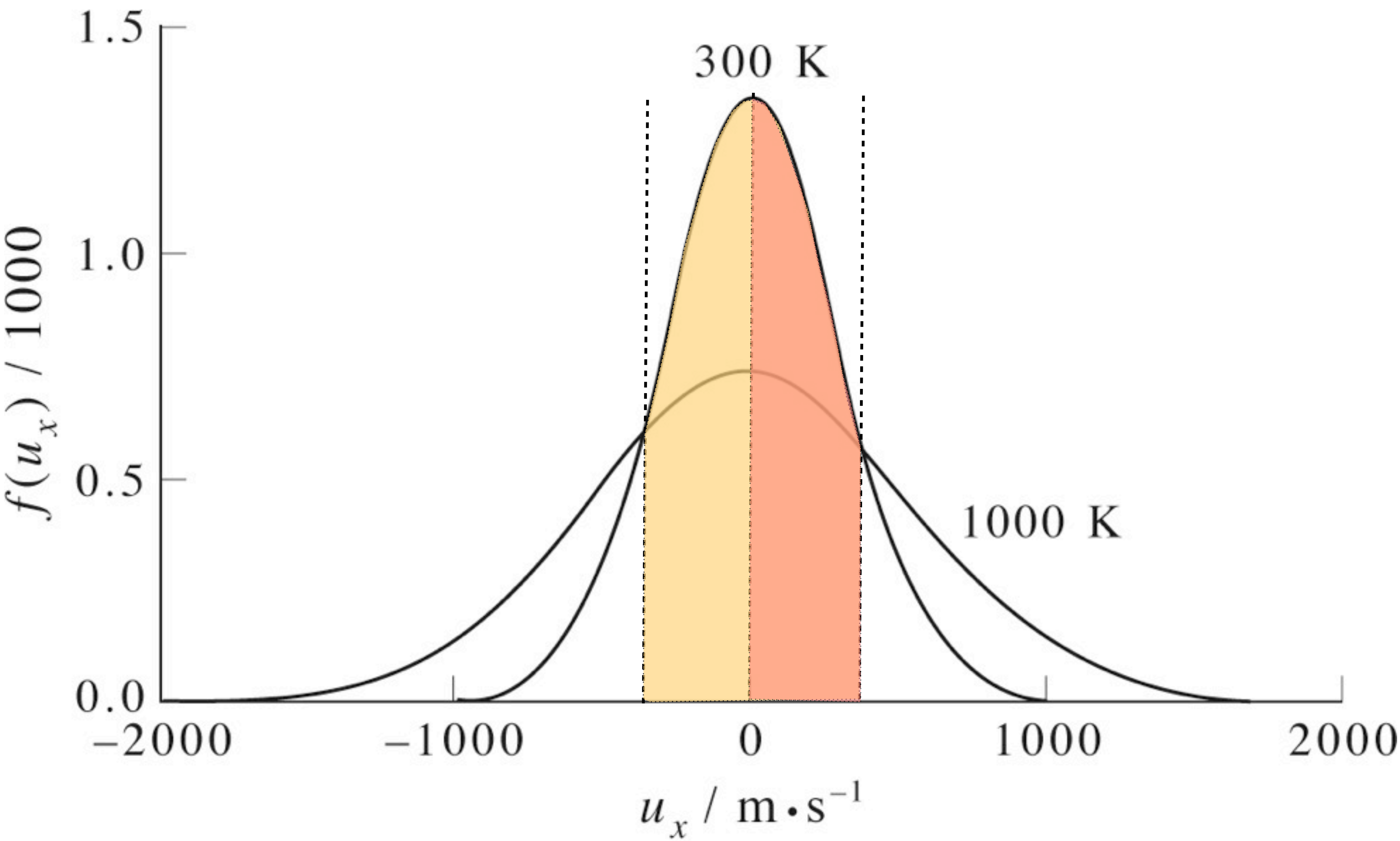
1.5
1.0
0.5
0.0
f(u_x) / 1000
300 K
1000 K
–2000
–1000
0
1000
2000
u_x / m•s^–1


1.5
1.0
0.5
0.0
300 K
1000 K
f(u_x) / 1000
−2000
−1000
0
1000
2000
u_x / m•s^−1



1.5
1.0
0.5
0.0
f(u_x) / 1000
300 K
1000 K
−2000
−1000
0
1000
2000
u_x / m•s^−1


1.5
1.0
0.5
0.0
300 K
1000 K
–2000
–1000
0
1000
2000
$f(u_x)$ / 1000
u_x / m•s^{-1}


1.5
1.0
0.5
0.0
f(u_x) / 1000
300 K
1000 K
–2000
–1000
0
1000
2000
u_x / m•s^–1

解析階がない　数値解を求めるしかない

$$\int_{-u_{x0}}^{u_{x0}} f(u_x)du_x = \sqrt{\frac{m}{2\pi k_{\mathrm{B}}T}} \int_{-u_{x0}}^{u_{x0}} \exp(-\frac{mu_x^2}{2k_{\mathrm{B}}T})du_x$$

$$= 2\sqrt{\frac{m}{2\pi k_{\mathrm{B}}T}} \int_{0}^{u_{x0}} \exp(-\frac{mu_x^2}{2k_{\mathrm{B}}T})du_x$$

$$w^2 = \frac{mu_x^2}{2k_{\mathrm{B}}T}, \quad w = \sqrt{\frac{m}{2k_{\mathrm{B}}T}}u_x, dw = \sqrt{\frac{m}{2k_{\mathrm{B}}T}}du_x$$

$$w_0^2 = \frac{mu_{x0}^2}{2k_{\mathrm{B}}T}, w_0 = \sqrt{\frac{m}{2k_{\mathrm{B}}T}}u_{x0}$$

$$\int_{-u_{x0}}^{u_{x0}} f(u_x)du_x = 2\sqrt{\frac{m}{2\pi k_{\mathrm{B}}T}}\sqrt{\frac{2k_{\mathrm{B}}T}{m}} \int_{0}^{w_0} e^{-w^2}dw$$

$$= \frac{2}{\sqrt{\pi}} \int_{0}^{w_0} e^{-w^2}dw$$

$$u_{x0} = \sqrt{\frac{2k_{\mathrm{B}}T}{m}}$$

$$w_0 = \sqrt{\frac{m}{2k_{\mathrm{B}}T}}u_{x0} = \sqrt{\frac{m}{2k_{\mathrm{B}}T}}\sqrt{\frac{2k_{\mathrm{B}}T}{m}} = 1$$

$$\int_{-u_{x0}}^{u_{x0}} f(u_x)du_x = \frac{2}{\sqrt{\pi}}\int_0^1 e^{-w^2}dw = 0.84270$$